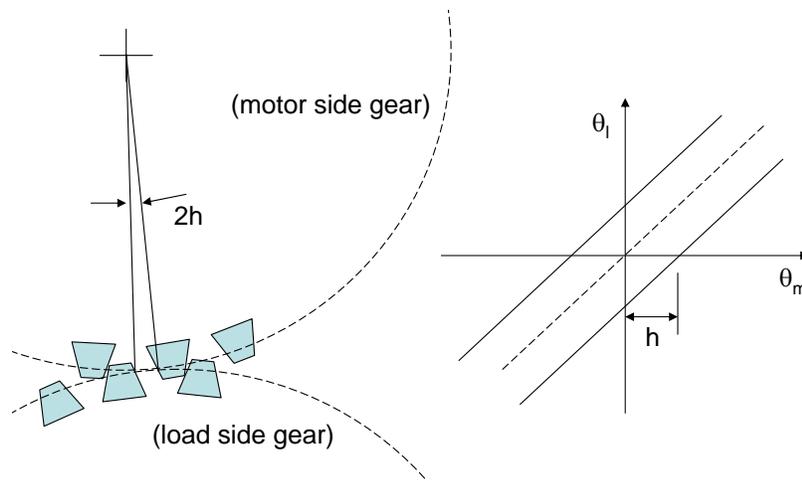


32 Motor Servo with Backlash

Consider the following: a motor is attached to a load through a gearbox, and the gearbox has a backlash behavior. Physically, backlash is a decoupling of gear teeth (due to imprecise meshing), so that the motor-side and the load-side teeth are not always in contact. Contacts are characterized by the condition $|\theta_m - \theta_l| > h$, where the θ 's are the motor and load angles respectively, and h is the half-width of the backlash. During contact, a small deflection material occurs according to a linear spring model. In the forward direction, that is, when θ_m leads θ_l by a positive amount,

$$\tau_t = k(\theta_m - \theta_l - h) \text{ when } \theta_m - \theta_l > h,$$

where τ_t is the torque transmitted from the motor side to the load. A similar condition holds in the reverse direction. Conversely, whenever θ_m is within h of the angle θ_l , the teeth are out of contact and no torque is passed between the motor and the load.



The rotary mass on the motor side is J_m ; on the load side, we have rotary mass J_l as well as a rotary damping b . It is desired to position the load via a servo, so a simple feedback controller is implemented of the form $\tau_m = -g\theta_l$, where τ_m is the motor torque. We assume that we can control the torque in the motor directly through a current amplifier.

All the physical properties in a particular system built by Company XYZ are well understood, *except* for the load-side damping b . This varies slowly over time because the bearings break in and then age, and because the load side is connected to an unpredictable environment. For the purposes of this problem, we assume that b is a random variable, described by a uniform probability density function, with minimum value zero and maximum value $0.02 \text{ Nm}/(\text{rad/s})$. The other parameters in the model are

```
Jm = 0.003 ; % motor-side inertia, kg-m^2
Jl = 0.01 ; % load-side effective inertia, kg-m^2
```

```

k = 10000 ;    % stiffness of tooth contact point, Nm/rad
h = 0.02 ;    % backlash half-width, rad
g = 6 ;       % position feedback gain, Nm/rad

```

The standard transient response that we are interested in has the initial condition of zero speeds, zero motor position, and a load position of h . We take as characteristic outputs of any particular model run the two-norms of the load-side position error, and the commanded torque. The two-norm of a signal over a time interval $[t_1, t_2]$ is defined as:

$$\|z(t)\|_2 = \sqrt{\int_{t_1}^{t_2} z^2(t) dt}$$

These norms are to be computed over the time range of 2.5-5.0 seconds after the feedback system turns on.

Question: What are the means and standard deviations of the two characteristic outputs defined above?

The attached code models the dynamic behavior, and runs a number of cases along the uniformly-divided range of damping b . I used a trapezoid rule for the integration of the mean, and just computed the straight standard deviation of the samples. It would also have been possible to make these calculations with a Gauss quadrature or a Monte Carlo technique.

A typical response is shown below versus time, as is the norm $\|z(t)\|_2$ versus b . The mean and standard deviation of the load-side position are 0.0101 and 0.00527 rad, respectively. The mean and standard deviation of the motor torque are 0.0609 and 0.0316 Nm, respectively.

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Backlash sensitivity problem

```

```

% FSH MIT Mechanical Engineering April 2008

```

```

clear all;
global Jm J1 k h g b ;

```

```

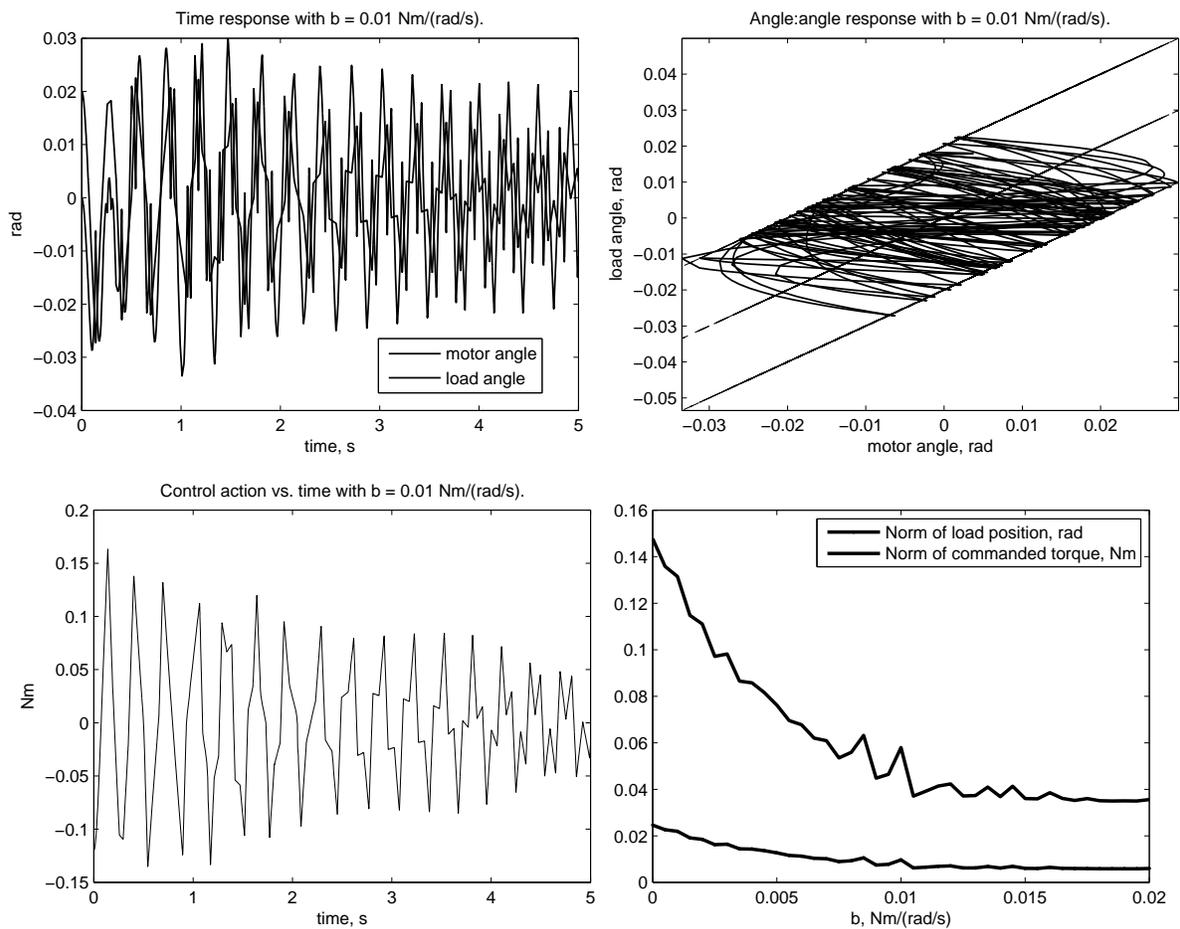
Jm = 0.003 ; % motor-side inertia, kg-m^2
J1 = 0.01 ; % load-side inertia, kg-m^2
k = 10000 ; % stiffness of tooth contact point, Nm/rad
h = 0.02 ; % backlash half-width, rad
g = 6 ; % position feedback gain, Nm/rad

```

```

bvec = 0:.0005:.02 ; % vector of dampings to apply,
                    % on load side, Nm/(rad/s) = kg*m^2/s

```



```

tfinal = 5 ; % final simulation time
tcut = 2.5; % time after which we will computing the signal norm
plotFlag = 0 ; % set to one to get all plots of trajectories

ct = 0 ; % counter
for b = bvec, % loop through all the b's

    disp(sprintf('Approx. closed-loop damping ratio: %g', ...
        b/2/sqrt((Jm+Jl)*g) ));

    % run the simulation - note initial error in load position
    % equal to the backlash half-width (states are listed in deriv.)
    [t,y] = ode45('backlashderiv',tfinal,[0 0 0 h]);

    tauMotor = -g*y(:,4); % recreate the motor torque command

    if plotFlag | b == bvec(end)/2, % show some intermediate results

```

```

    figure(1);clf;hold off;
    subplot('Position',[.2 .2 .5 .5]);
    plot(t,y(:,[2,4]),'LineWidth',1);
    xlabel('time, s');
    ylabel('rad');
    legend('motor angle', 'load angle',4);
    title(sprintf('Time response with b = %g Nm/(rad/s).', b));

    figure(2);clf;hold off;
    subplot('Position',[.2 .2 .5 .5]);
    plot(y(:,2),y(:,4),'r','LineWidth',1);
    hold on;
    plot(y(:,2),y(:,2),'k--',y(:,2),y(:,2)+h,'k--',...
        y(:,2),y(:,2)-h,'k--');
    axis('tight');
    xlabel('motor angle, rad');
    ylabel('load angle, rad');
    title(sprintf('Angle:angle response with b = %g Nm/(rad/s).', b));

    figure(3);clf;hold off;
    subplot('Position',[.2 .2 .5 .5]);
    plot(t,tauMotor);
    xlabel('time, s');
    ylabel('Nm');
    title(sprintf('Control action with b = %g Nm/(rad/s).', b));

end;

ct = ct+1 ;

% calculate the norm of the load motion and the torque, after tcut
[dum,i] = sort(abs(t-tcut)) ;
energyNorm(ct) = sqrt(trapz(t(i(1):end),y(i(1):end,4).^2));
tauMotorNorm(ct) = sqrt(trapz(t(i(1):end),tauMotor(i(1):end).^2));

disp(sprintf('%d/%d done.', ct,length(bvec)));

end;

figure(4);clf;hold off;
subplot('Position',[.2 .2 .5 .5]);
plot(bvec,energyNorm,'.-',bvec,tauMotorNorm,'LineWidth',2);
legend('Norm of load position, rad','Norm of commanded torque, Nm');

```


MIT OpenCourseWare
<http://ocw.mit.edu>

2.017J Design of Electromechanical Robotic Systems
Fall 2009

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.