

19 Aliasing

Consider sampling the sinusoidal process: $x(t) = 0.9 \sin(2\pi f_o t)$.

1. When we sample the signal at frequency f_s (Hz) or ω_s (rad/s), the Nyquist rate of $f_N = f_s/2$ is the maximum frequency the sampling can detect. See the figure for some examples; undersampling of 1.7 indicates that the process frequency exceeds the Nyquist rate by a factor of 1.7. What is the required sampling rate to detect $f_o = 20\text{kHz}$? Extrapolating from the figure, is this sampling rate good enough to observe the process with reasonable fidelity?

The required rate is twice the process rate, or 40kHz. This rate does not give good phase or amplitude information of the 20kHz signal; about 200-400kHz would be much better, giving five to ten points per cycle.

2. Suppose that you can only sample at $f_s = 5.5\text{kHz}$. What is the time step? For the time interval $t = [0 \ 0.002]$, plot $x(t)$ sampled at a high enough rate that you can clearly see the sine wave, and overlay it with the signal sampled at f_s . What is the apparent (*aliased*) frequency of the sampled signal?

A simple rule to predict this aliased frequency is: decrement f_o by f_s enough times to get within the observable frequency range of $[-f_N, f_N]$. The absolute value of this result is the aliased frequency.

Sampling at 5.5kHz gives a time step of 0.182 milliseconds. From the top subplot in Figure 4, this gives 2kHz aliased. Applying the formula above, we find $20\text{kHz} - 4 \times 5.5\text{kHz} = -2\text{kHz}$.

3. Repeat the above analysis for $f_s = 6.5\text{kHz}$.

Sampling at 6.5kHz gives a time step of 0.154 milliseconds. From the middle subplot and the calculation, we get 0.5kHz.

4. Repeat the above analysis for $f_s = 7.5\text{kHz}$.

Sampling at 7.5kHz gives a time step of 0.133 milliseconds. From the bottom subplot and the calculation, this gives 2.5kHz aliased.

```
%-----
% Aliasing - explore several different undersampling sins.

% Franz Hover MIT Mechanical Engineering

clear all;

tfinal = .002 ;
```

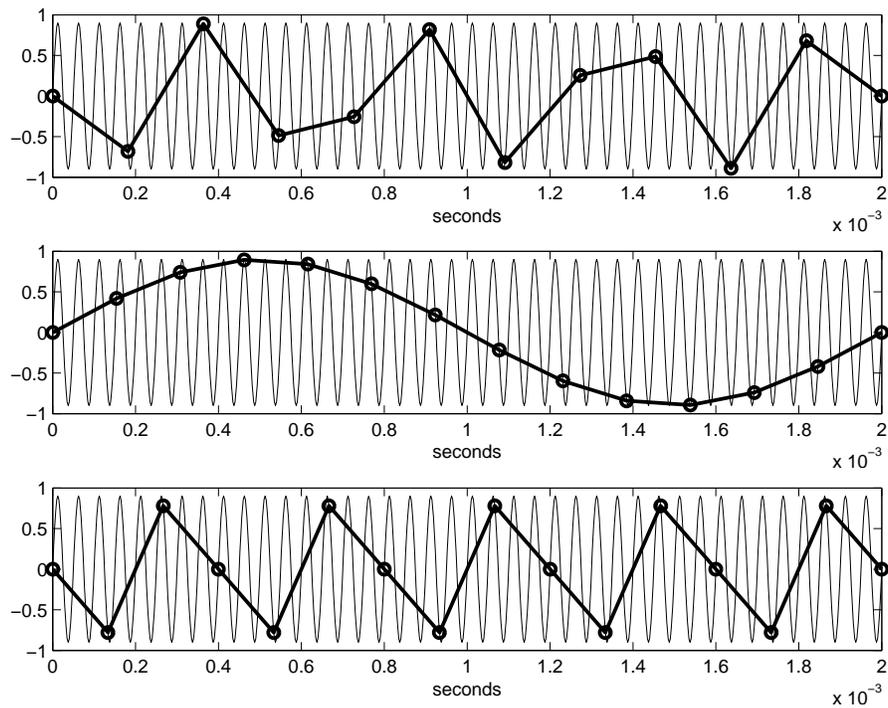


Figure 4: Aliasing: sampling a 20kHz signal at 5.5, 6.5, and 7.5kHz. A dangerous business!

```

w0 = 20000*2*pi;
ws = 7500*2*pi; % change the sampling rate here [5500 6500 7500]

dtfi = 2*pi/w0/20 ; % fine time scale
tfi = 0:dtfi:tfinal ;

dt = 2*pi/ws ; % sampling time scale
t = 0:dt:tfinal ;

xfi = .9*sin(w0*tfi) ; % here are the signals
x = .9*sin(w0*t) ;

figure(1);clf;hold off;
subplot(3,1,1);
plot(tfi,xfi);
hold on;
plot(t,x,'ro-','LineWidth',2);
xlabel('seconds');
disp('Save the figure if necessary.');
```


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