

1 Linear Time Invariance

1. For each system below, determine if it is linear or non-linear, and determine if it is time-invariant or not time-invariant (adapted from Siebert 1986).

(a) $y(t) = u(t + 1)$

The system is linear time-invariant; the output is just a time-advanced version of the input - it is noncausal!

(b) $y(t) = 1/u(t)$

Nonlinear, time-invariant. Replace $u(t)$ with $\alpha u(t)$ - this does not get us to $\alpha y(t)$, which would be required for linearity.

(c) $3\ddot{y} + \dot{y}(t) - y(t) = u(t)$

Linear time-invariant; an unstable second-order system. We have to assume $y(0) = 0$, and that we are talking about delays only if $u(t) = 0$ for $t \leq 0$.

(d) $y(t) = \sin(t)u(t)$

Linear time-varying. The coefficient $\sin(t)$ is a function of time, so if a given input trajectory is played with different starting times, the outputs will be different - unless the initial times are off by a integer multiple of 2π .

(e) $y(t) = u(t) + 2$

Nonlinear time-invariant. Tiny input $u(t)$ still gives an output of about two, whereas large inputs will give an output of about the same; hence, the system as written does not capture scaling of $u(t)$.

(f) $y(t) = \int_{-\infty}^t u(t_1)\sin(t - t_1)dt_1$

Linear time-invariant. Linearity is easy because the integral is a linear operator. Time-invariance is a little harder to show. Look at the right side first, with an advance in the input of τ , and where we set $t_2 = t_1 + \tau$:

$$\int_{-\infty}^t u(t_1 + \tau)\sin(t - t_1)dt_1 = \int_{-\infty}^{t+\tau} u(t_2)\sin(t - (t_2 - \tau))dt_2$$

Then advance the time in the expression for y :

$$y(t + \tau) = \int_{-\infty}^{t+\tau} u(t_1)\sin(t + \tau - t_1)dt_1$$

These are the same and hence the system is time-invariant.

2. Determine whether the following is a time-invariant system or not; why? A large rocket in early flight is steered in pitch and yaw with control fins. As you know, the rocket assembly burns a *huge* amount of fuel during takeoff, and the rate of this burn is not affected by the steering. Because of the burn, however, the mass of the rocket is continually decreasing and its distribution is continually changing. Consider that the input is a perturbation to the fin angle, and the output is the pitch angle of the rocket. (As described, this system is linear.)

The system is time-varying. The problem is an inverted pendulum, but the mass

at the top is changing; hence, a given deflection of the control surfaces will give a different dynamic response at different times during the flight. This attribute of the rocket requires a flight controller that also changes as an explicit function of time. Historically, the problem was one of the early successes in optimal control theory.

3. Determine whether the following is likely to be a linear system or not; why? A power electronics network contains resistors that heat up when the current through them is large. This heating causes them to increase their resistance. (As described, this system is time invariant.)

The system is nonlinear, because running at different power levels will lead to different operating temperatures and different resistances. This assumes there is an adequate heat sink!

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