

# Sample Problems

Massachusetts Institute of Technology  
Department of Mechanical Engineering

## 2.12 Introduction to Robotics

### **Mid-Term Examination**

October 27, 2004

2:30 pm – 4:30 pm

Close-Book. Two sheets of notes are allowed.

Show how you arrived at your answer.

Do not leave multiple answers. Indicate which one is your correct answer.

#### **Problem 1**

Shown below is a robotic arm rowing a boat at the Charles Regatta. For the sake of simplicity, we consider only planar motion, assuming that the two-link robot arm and the oar are constrained in a horizontal plane. The oar is free to rotate about the pin, called an oarlock, and the robot pulls the oar at the distance  $d$  from the oarlock. The robot endpoint is connected to the oar at point  $E$  through a friction-less passive joint. A hydraulic force  $f$  acts on the blade at a distance  $D$  from the oarlock. Assume that the hydraulic force is perpendicular to the oar. The oarlock is at coordinates  $(x_o, y_o)^T$  with reference to the base coordinate system attached to the first joint of the robot arm. Assume no friction and no gravity. Using the notation shown in the figure, answer the following questions.

a). Obtain the coordinates of the robot endpoint  $x_e, y_e$  as functions of joint angles  $\theta_1, \theta_2$ , and differentiate the endpoint coordinates with respect to joint angles in order to derive the Jacobian.

b). Let angle  $\alpha$  be the angle of the oar measured from the  $x$  axis. Obtain the endpoint coordinates  $x_e, y_e$  as functions of  $\alpha$ . Under what condition, is angle  $\alpha$  a generalized coordinate to locate the entire system?

c). Using the principle of virtual work, show that the joint torques  $\tau_1, \tau_2$  needed for pulling the oar against the hydraulic force  $f$  must satisfy the following relationship:

$$f = \frac{d}{D \sin(\theta_2 - \theta_1)} \left\{ \frac{\tau_1}{\ell_1} \sin(\theta_2 - \alpha) + \frac{\tau_2}{\ell_2} \sin(\theta_1 - \alpha) \right\}$$

d). DC motors are used for driving the two joints of the robot. Let  $K_{m1}$  be the motor constant of the first motor, and  $K_{m2}$  be that of the second motor. Show that the total power loss at the two motors when generating joint torques  $\tau_1, \tau_2$  is given by:

$$P_{loss} = \frac{\tau_1^2}{r_1^2 K_{m1}^2} + \frac{\tau_2^2}{r_2^2 K_{m2}^2}$$

where  $r_1, r_2$  are gear ratios of the motors. (Note that the second motor is fixed to the base and its output torque is transmitted to the second joint, just like the 2.12 lab manipulator.)

e). Obtain the optimal values of the joint torques  $\tau_1, \tau_2$  that minimize the total power loss in both motors,  $P_{\text{loss}}$ , while bearing the hydraulic force  $f$  at the configuration:

$\theta_1 = 45^\circ, \theta_2 = 135^\circ, \alpha = 0$ . Assume that the link dimensions are  $\ell_1 = \ell_2 = \frac{1}{\sqrt{2}}, d = D$

and that the products of the motor constant and the gear ratio are  $r_1 K_{m1} = 1, r_2 K_{m2} = 0.5$ , in dimensionless form.

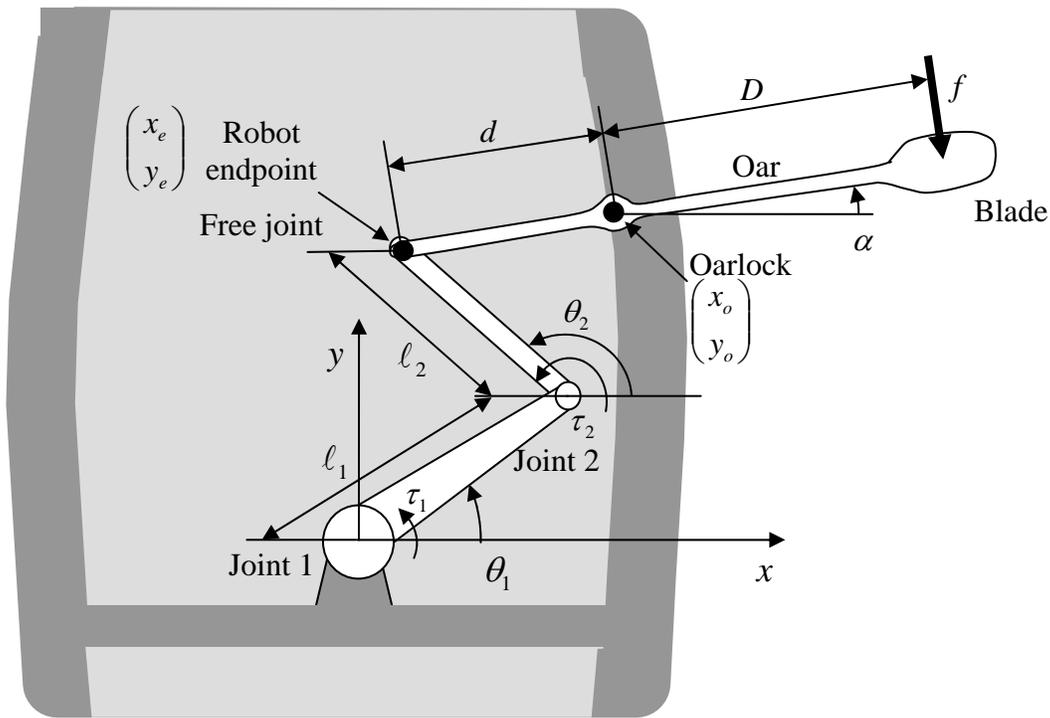


Figure 1 Two d.o.f. robot rowing a boat

## Problem 2

Shown below is a robot arm with three revolute joints. Coordinate system  $O - xyz$  fixed to the base link, *Link 0*, represents the Cartesian coordinates of the endpoint  $x_e, y_e, z_e$ . Joint angle  $\theta_1$  is measured about the joint axis  $OA$  ( $z$  axis) from axis  $x$  to line  $OB'$ , where point  $B'$  is the projection of point  $B$  onto the  $xy$  plane. Another coordinate system  $B - uvw$  is placed at point  $B$  in such a way that the  $u$  and  $w$  axes are parallel to the  $xy$  plane and that the  $v$  axis is parallel to the  $z$  axis. The second joint axis  $AB$  is horizontal, and joint angle  $\theta_2$  is measured from axis  $u$  to line  $BD$ . Joint angle  $\theta_3$  is measured about the joint axis  $CD$  from line  $BD$  to *Link 3*, i.e. line  $DE$ . Link dimensions are  $OA = \ell_0, AB = \ell_1, BD = \ell_2$ , and  $DE = \ell_3$ . (For the purpose of explaining the

kinematic structure, points C and D are shown to be different, but they are the same point, i.e. the length CD is zero.) Note also that  $\angle OAB = \angle ABD = 90^\circ$ . Answer the following questions.

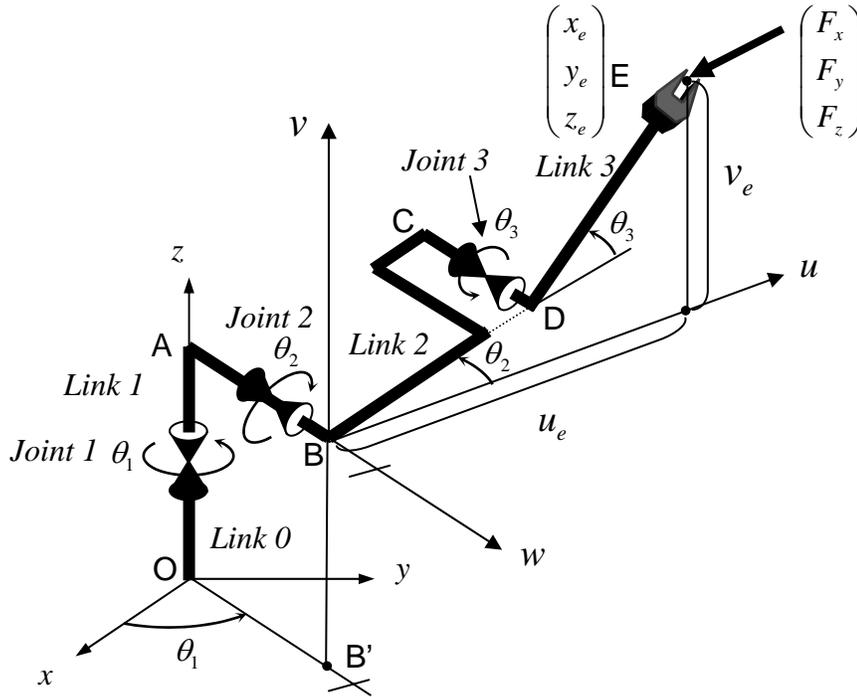


Figure 1 Kinematic structure of a 3 d.o.f. robot

- Obtain the coordinates of the endpoint  $E$  viewed from the frame,  $B-uvw$ , that is,  $u_e, v_e$  in the figure.
- Obtain the endpoint coordinates  $x_e, y_e, z_e$  viewed from the base coordinate system  $O-xyz$ .
- For a given endpoint position, how many solutions exist to the inverse kinematics problem? Sketch all the different configurations leading to the same endpoint coordinates.
- Obtain the 3x3 Jacobian matrix relating the endpoint velocities  $\dot{x}_e, \dot{y}_e, \dot{z}_e$  to joint velocities  $\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3$ .
- Forces  $F_x = 10 \text{ N}, F_y = 0, F_z = 0$  act at the endpoint, when the joint angles are  $\theta_1 = 0^\circ, \theta_2 = 45^\circ, \theta_3 = 90^\circ$ . Assume that  $\ell_2 = \ell_3$ . Obtain the joint torques needed for bearing the force acting at the endpoint,  $F_x = 10 \text{ N}, F_y = 0, F_z = 0$ . Discuss the physical sense of the result.

The following question is for your extra credit. If you have time, work on it.

- Obtain the joint angles of all the singular configurations by solving the determinant condition of the Jacobian matrix. Sketch singular configurations, and determine in which direction the endpoint cannot be moved with a non-zero velocity.