Introduction to Computers and Programming

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Lecture 18 May 3 2004

Four Variable K-Maps Example-2

Using a 4-variable K-Map, simplify the following Truth table

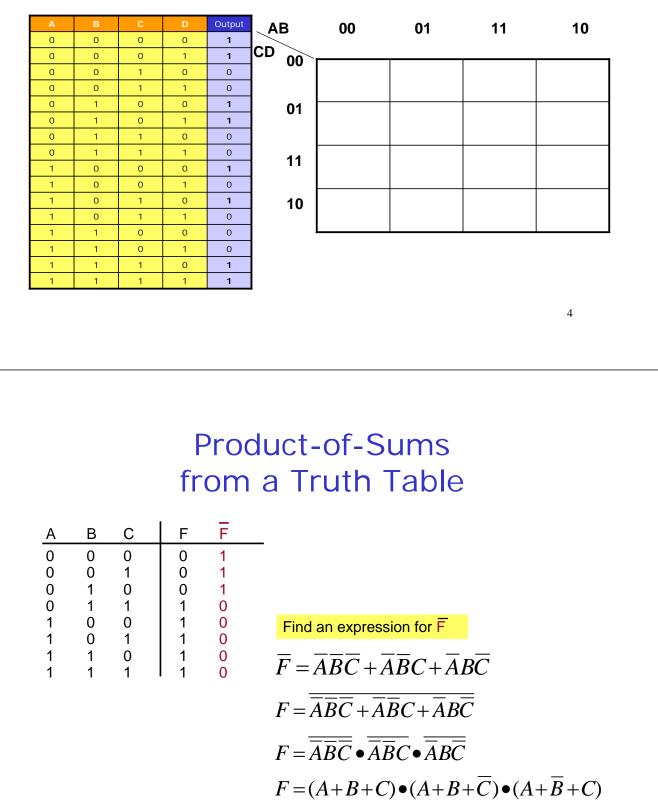
А	В	С	D	Output	
0	0	0	0	0	
0	0	0	1	0	CD
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	1	
0	1	0	1	0	
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	1	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	0	
1	1	1	0	1	
1	1	1	1	0	

AB D	00	01	11	10
00	0	1	0	0
01	0	0	0	1
11	0	0	0	0
10	0	0	1	0

$$Output = \overline{A}B\overline{C}\overline{D} + A\overline{B}\overline{C}D + ABC\overline{D}$$

Four Variable K-Maps Example-2

Using a 4-variable K-Map, simplify the following Truth table



Maxterms

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• (
-

- $F = (A + B + C) \bullet (A + B + \overline{C}) \bullet (A + \overline{B} + C)$
- To find a Product-of-Sums form for a truth table
 - Make one maxterm for each row in which the function is zero
 - For each maxterm, each variable appears once
 - In its complemented form if it is one in the row
 - In its regular form if it is zero in the row

Today

- Propositional Logic
- From English to propositions
- Quantified statements
- Tomorrow: Methods of proving theorems

Propositional Logic

- Logic at the sentential level
 - Smallest unit: sentence
 - Sentences that can be either true or false
 - This kind of sentences are called Propositions
- If a propositions is true, then its truth value is "true", if proposition if false, then the truth value is "false"

Propositional Logic

- The following are propositions:
 - Grass is green
 - -2 + 4 = 4
- The following are not propositions:
 - Wake up
 - Is it raining today?
 - -X > 2
 - -X = X

Elements of Propositional Logic

Connectives

not	-
and	^
or	\sim
if_then (implies)	\rightarrow
iff	\leftrightarrow

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Connectives

Ρ	Q	(P ∨ Q)	
F	F	F	
F	Т	Т	
Т	F	Т	
Т	Т	Т	

Ρ	¬ P	
Т	F	
F	Т	

Ρ $(P \land Q)$ Q F F F F Т F F Т F т Т Т

Ρ	Q	$(P\toQ)$
F	F	Т
F	Т	Т
Т	F	F
Т	Т	Т

Q	$(P\leftrightarrowQ)$
F	Т
Т	F
F	F
Т	Т
	F

Truth tables

Concept Question Given P \rightarrow Q, Is Q \rightarrow P True?

1. Yes

2. No

3. I don't know

4. What is P->Q

Converse and Contrapositive

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• For $P \rightarrow Q$	
$Q \rightarrow P$	is called its converse
¬Q → ¬P	is called its contrapositive

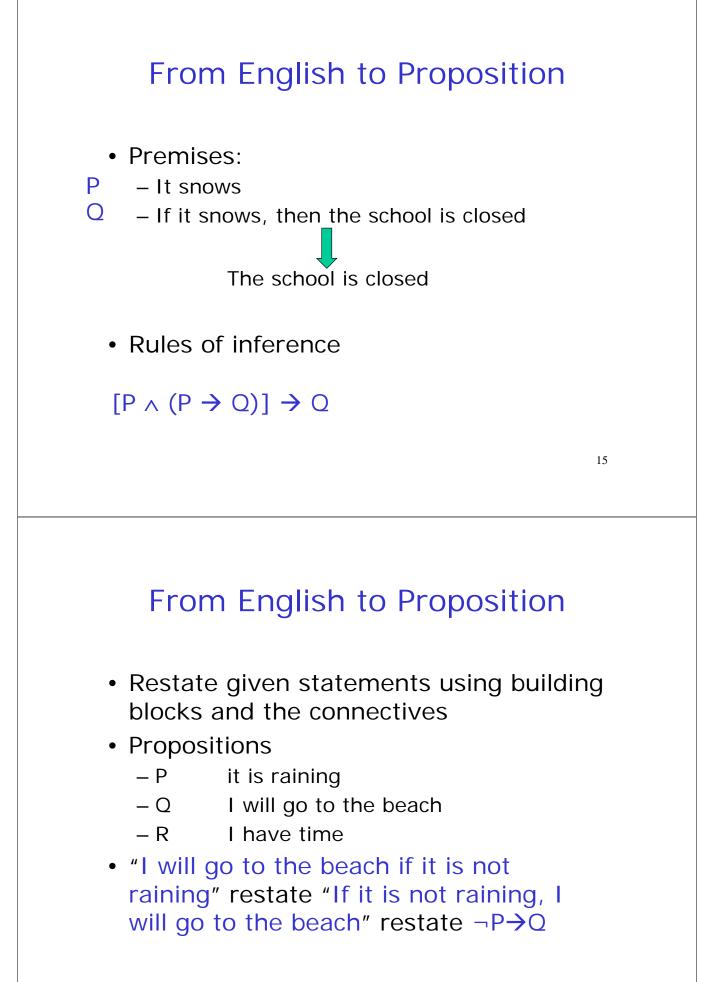
Example: If it rains, then I get soaked

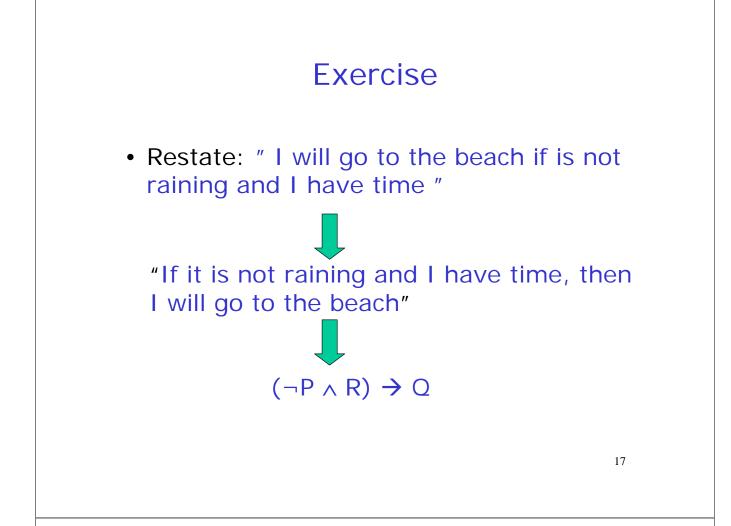
converse :

If I get soaked, then it rains

contrapositive :

If I don't get soaked, then it does not rain





Rule of Inference: Modus Ponens

 $(\mathbf{p} \land (\mathbf{p} \rightarrow \mathbf{q})) \rightarrow \mathbf{q}$ is a **tautology**. It states that if we know that both an *implication* $\mathbf{p} \rightarrow \mathbf{q}$ is true and that its *hypothesis*, **p**, is true, then the *conclusion*, **q**, is true.

Ex: Suppose the *implication* "If the bus breaks down, then I will have to walk" and its hypothesis "the bus breaks down" are true.

Then by modus ponens it follows that "I will have to walk".

Ex: Assume that the *implication* $(n > 3) \rightarrow (n^2 > 9)$ is true. Suppose also that n > 3. Then by *modus ponens*, it follows that $n^2 > 9$.

Fallacy: Affirming the Conclusion

 $(\mathbf{q} \land (\mathbf{p} \rightarrow \mathbf{q})) \rightarrow \mathbf{p}$ is a **contingency**. It states that if we know that both an implication $\mathbf{p} \rightarrow \mathbf{q}$ is true and that its conclusion, \mathbf{q} , is true, then the hypothesis, \mathbf{p} , is true.

Ex: Suppose the implication "If the bus breaks down, then I will have to walk" and its conclusion "I will have to walk" is true. It **does not** follow that the bus broke down. Perhaps I simply missed the bus.

Ex: Consider the implication $(n > 3) \rightarrow (n^2 > 9)$ which is true. Suppose also that $n^2 > 9$. It does not follow that n > 3. It might be that n = -4 for example.