16.06 Principles of Automatic Control
Lecture 28

The Nichols Chart

The Nichols chart may be thought of as a Nyquist plot on a log scale. A Nyquist plot is a plot in the complex plane of

$$G(j\omega) = \underbrace{\mathrm{Re}(G(j\omega))}_{\text{x-coordinate}} + \underbrace{j\mathrm{Im}(G(j\omega))}_{\text{y-coordinate}}$$

Instead, on a Nichols chart, we plot

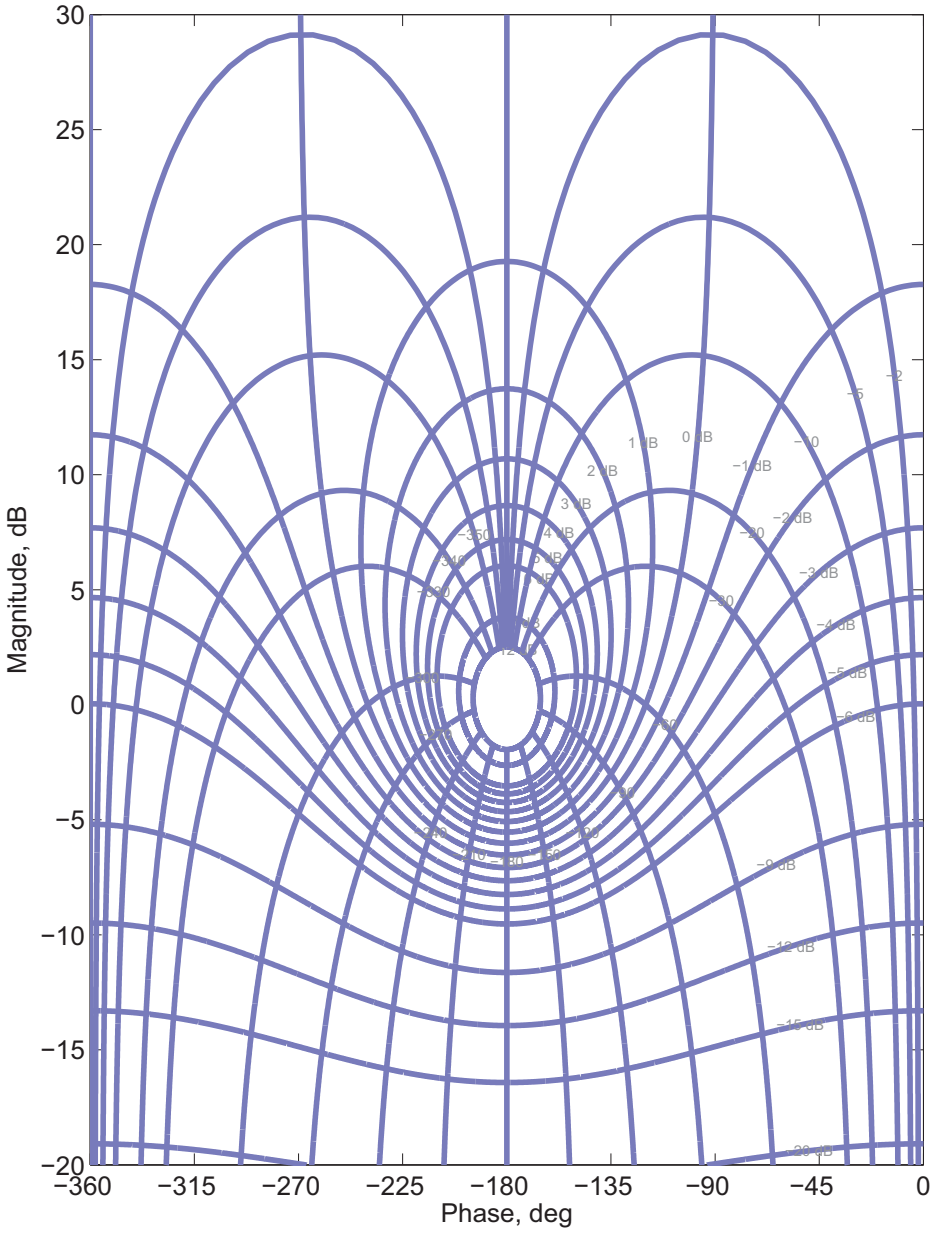
$$\log\ G(j\omega) = \underbrace{\log|G(j\omega)|}_{\text{y-coordinate}} + j\underbrace{\angle(G(j\omega))}_{\text{x-coordinate}}$$

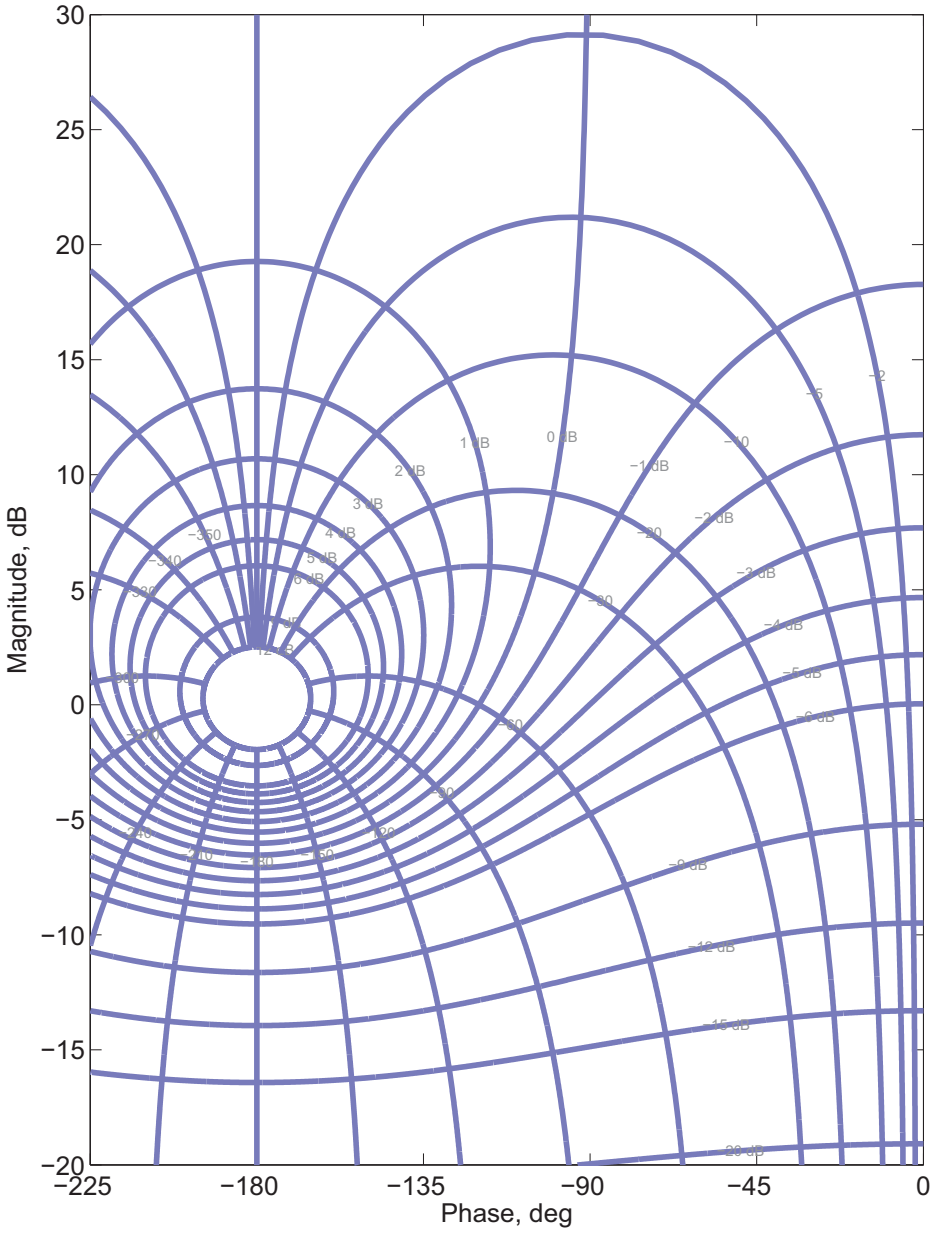
Notice that we reverse the coordinates - the real part is plotted on the vertical, and the imaginary part is plotted on the horizontal.

In addition, the chart has contours of constant closed-loop magnitude and phase,

$$M = \left|\frac{G}{1+G}\right|$$
$$N = \angle\frac{G}{1+G}$$

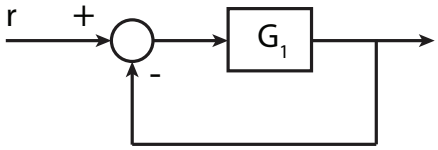
The Nichols chart template is shown below. Usually, we are interested in the range of frequencies where the phase is greater than -180°. The Nichols chart is often expanded (see plot below).


30
25
20
15
10
5
0
−5
−10
−15
−20
Magnitude, dB
−360
−315
−270
−225
−180
−135
−90
−45
0
Phase, deg


30
25
20
15
10
5
0
−5
−10
−15
−20
−225
−180
−135
−90
−45
0
Magnitude, dB
Phase, deg

The Nichols chart was once very useful, since computers were not available to do the kids of calculations that are now done by e.g., Matlab.

However, Nichols chart may be used to give insight into the closed-loop behavior of systems. Consider first the system

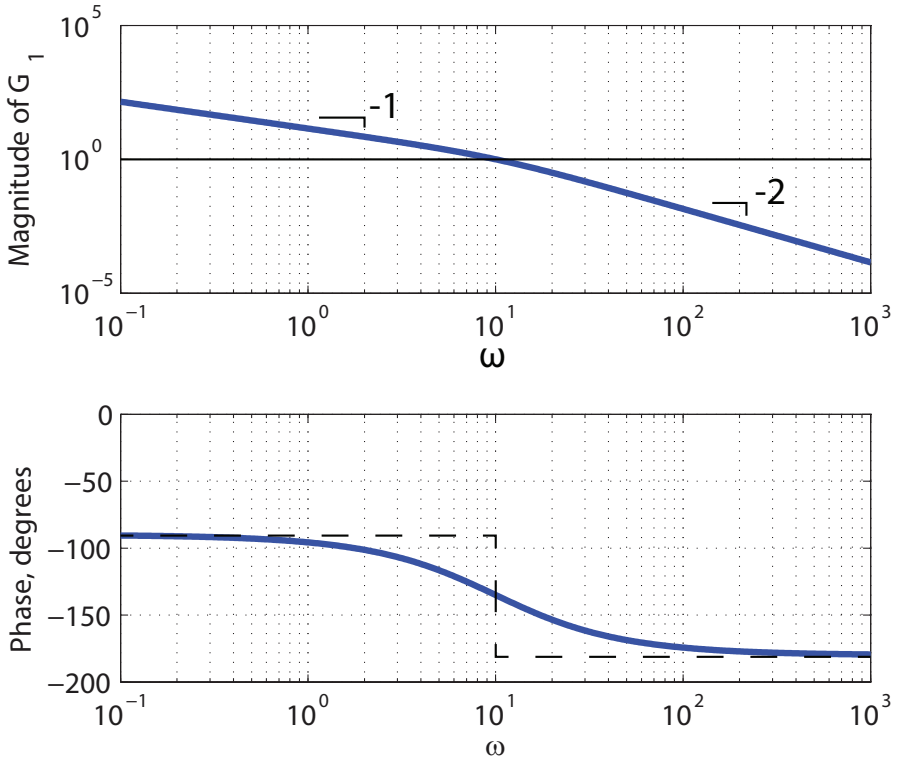


where

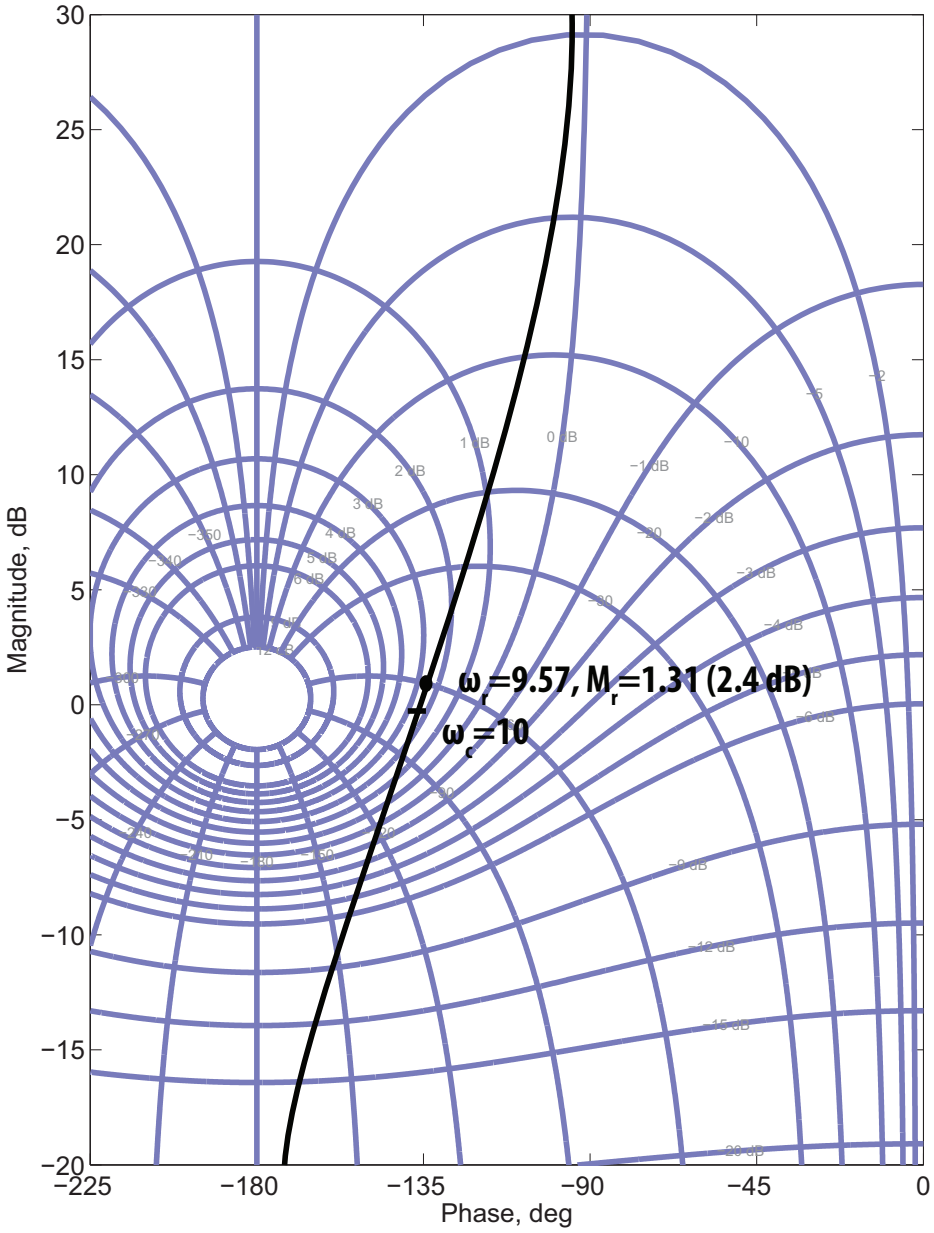
$$G_1 = \frac{\sqrt{2}\,10}{s(1+s/10)}$$

which has $\omega_c = 10$ r/s, $PM = 45^\circ$.

Bode of G_1:



The Nichols plot can be made by lifting points of the Bode plot, at individual frequencies, and plotting on the Nichols chart. See plot below for the plot of G_1 :



Note that $\omega_r \approx \omega_c$, so the peak in the frequency response (CL) is very close to crossover. Note also that

$$M_p = 0.23$$

For PM = 45°, we expect

$$\begin{aligned} \zeta &= 0.45 \\ \Rightarrow M_p &= 0.21 \\ M_r &= 1.24 \end{aligned}$$

In pretty good agreement with the actual results.

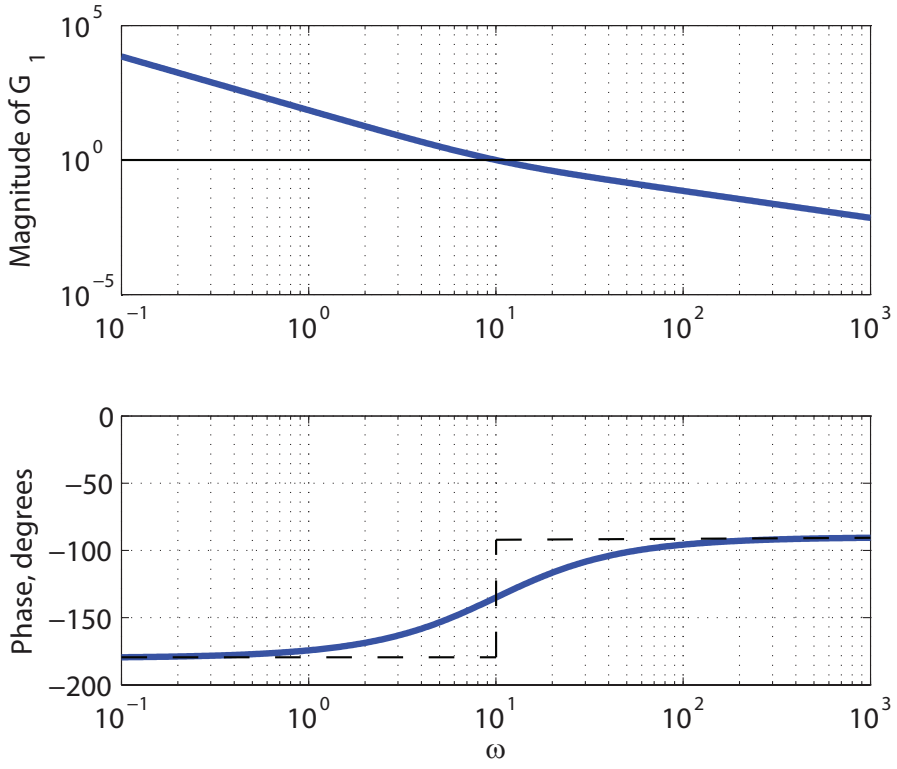
Now consider the plant

$$G_2 = \frac{100}{\sqrt{2}} \frac{1 + s/10}{s^2}$$

in a similar unity feedback control. For this system, we have

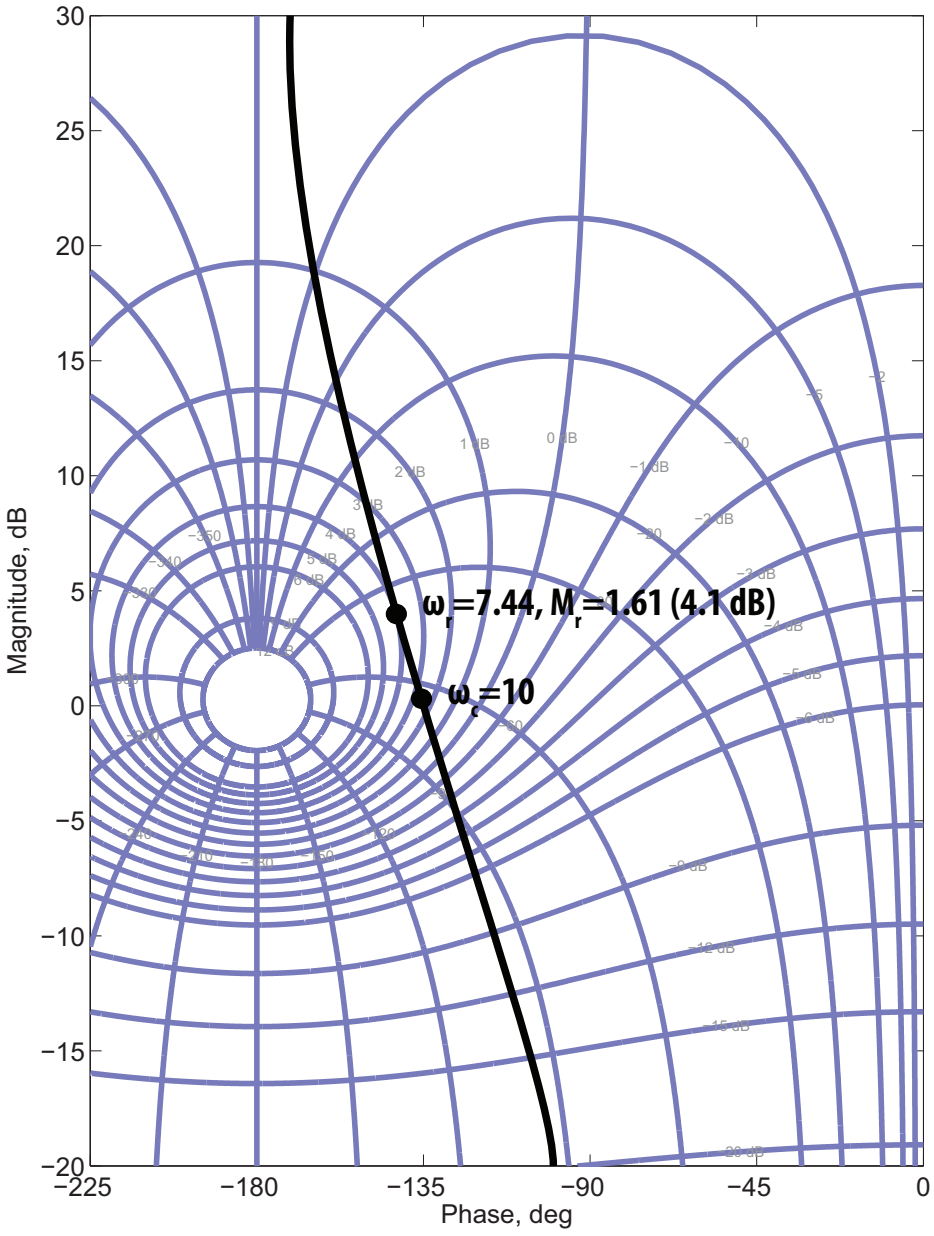
$$\text{PM} = 45^\circ,\ \omega_c = 10\,\text{r/s},\ \textit{also.}$$

Bode plot:



Since the crossover and phase margin are the same, we expect to get similar performance. Do we?

One clue can be seen in the Nichols chart, below.



Note that, in this case, ω_r is significantly smaller than ω_c, and M_r is larger than might be

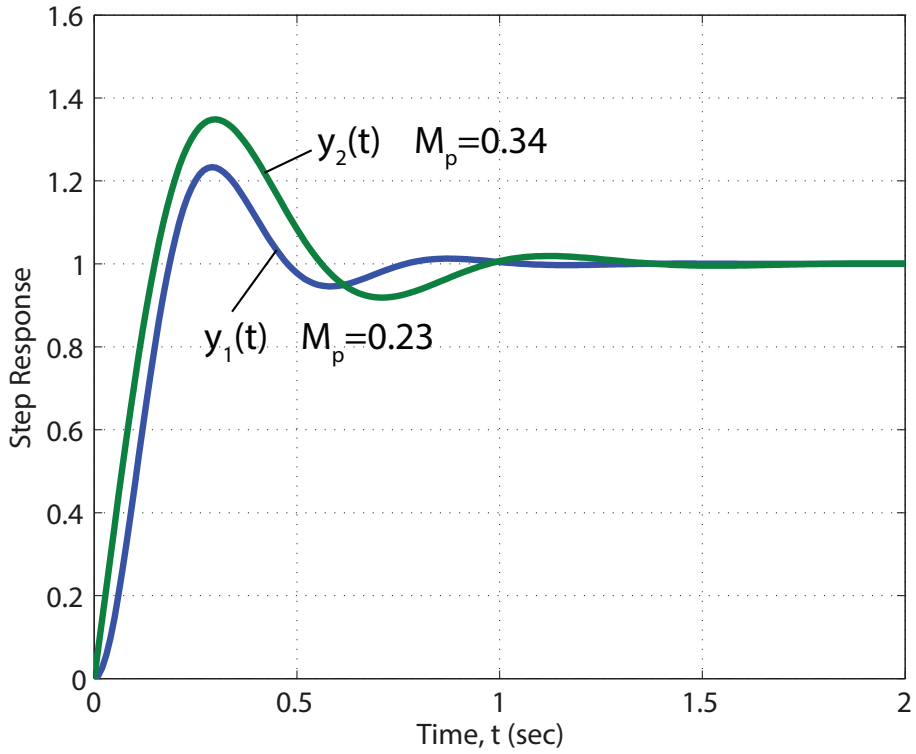
expected from the PM. So we would expect that the closed-loop system

$$T_2 = \frac{G_2}{1 + G_2}$$

would be a bit slower, and have more overshoot, than the system

$$T_1 = \frac{G_1}{1 + G_1}$$

even though they have the same PM and ω_c.
In fact, this is the case, as seen from the step responses below.



Counting Encirclements on a Nichols Chart

Counting encirclements on a Nichols chart can be tricky, because

1. The $-1/k$ point can be on either the $-180°$ line **or** the $0°$line.

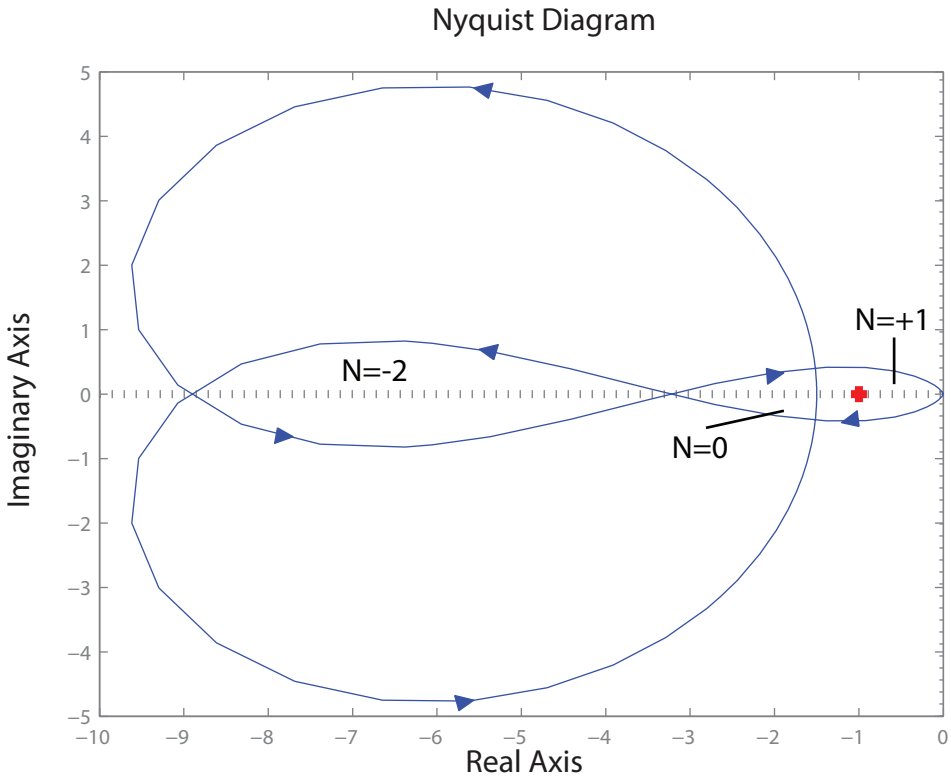
2. CW and CCW are reversed, because the orientation of the axes is reversed.

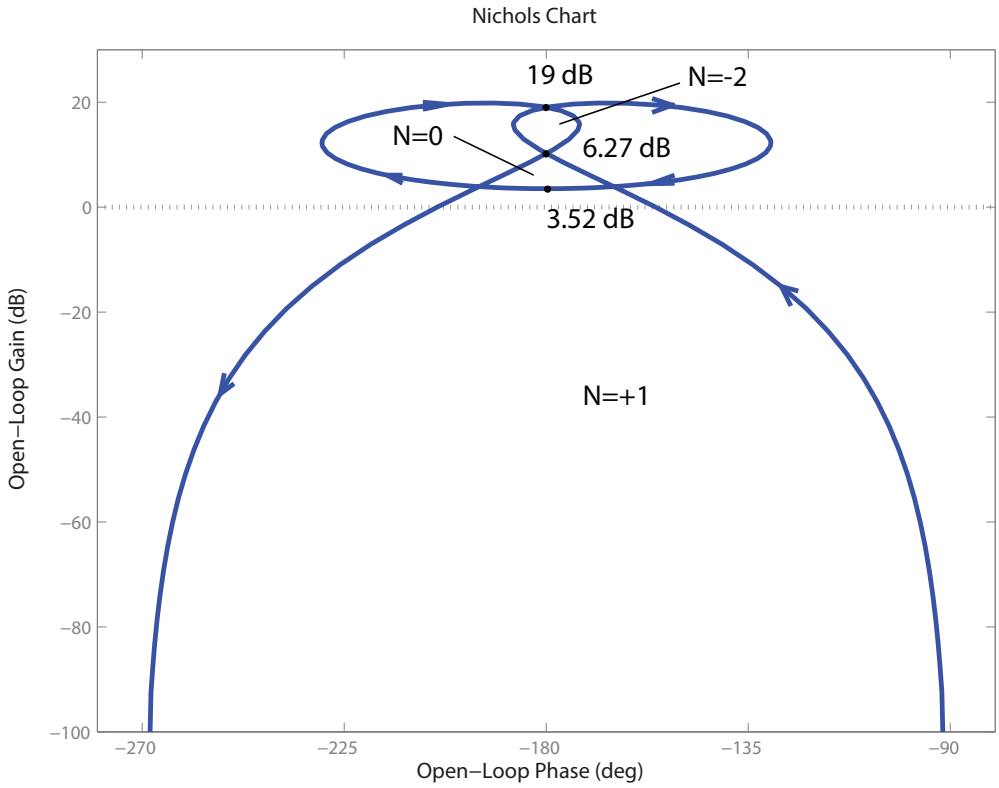
Will demonstrate with examples.

Example 1.

$$G(s) = 3000\,\frac{s-0.1}{(s-1)(s-2)(s+10)^2}$$

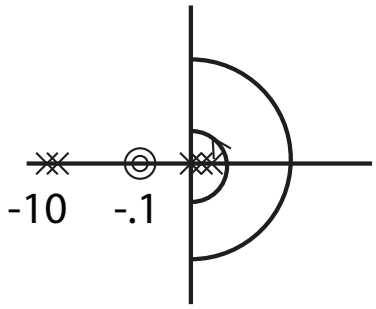
Nyquist and Nichols plots are shown below.



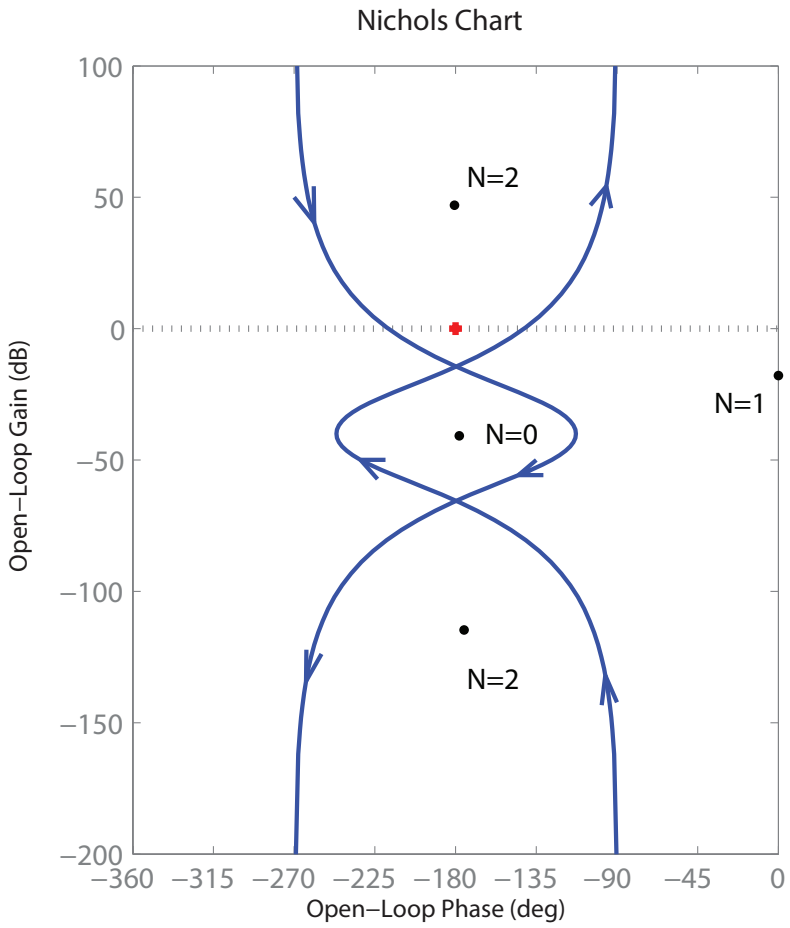


Example 2.

$$G(s) = \frac{1}{s^3}\frac{(s+0.1)^2}{(s+10)^2}$$



Nichols chart is shown below. Note that care must be used to properly close contour near $\omega = 0$.


Nichols Chart
100
50
0
−50
−100
−150
−200
−360
−315
−270
−225
−180
−135
−90
−45
0
Open−Loop Gain (dB)
Open−Loop Phase (deg)
N=2
N=1
N=0
N=2

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