16.06 Principles of Automatic Control
Lecture 35

Higher Harmonic Control

Helicopters and other rotating machinery often have serious vibration at multiples of the rotation speed, or harmonics.

To eliminate vibration, need to have

$$S(s) = \frac{1}{1 + K(s)G(s)}$$
(sensitivity transfer function)

be zero at $S = jN\Omega$,

where

Ω = roatation rate in r/s

N = harmonic to be controlled.

Therefore, need to have

$$K(jN\Omega) = \infty$$

To achieve this, K must have an oscillator at $\omega_n = N\Omega,\ \zeta = 0$. So

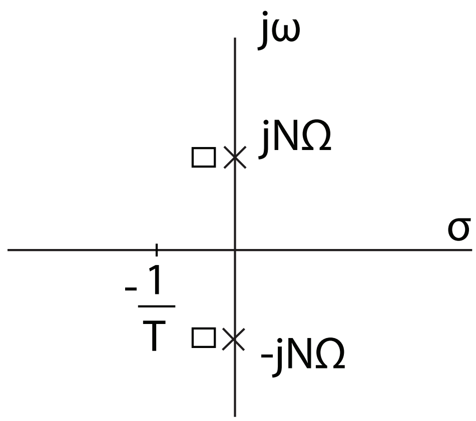
$$K(s) \approx \frac{k}{s^2 + (N\Omega)^2}$$

But usually need a zero as well, because we want the pole on the $j\omega$ axis to move directly left, at a departure angle of 180°.

Often, "low gain" control in OK, so the time constant T of the system can be long. That is, the closed-loop poles associated with the oscillator will be at

$$S = -\frac{1}{T} \pm jN\Omega$$

where T is large, compared to $1/\Omega$.



For a given $G(s)$, how do we get poles at desired location?

We could use Root locus methods; however, we don't really know $G(s)$,we only know $G(j\omega)$.

Where are roots of $1 + K(S)G(s)$?

If we take

$$K(s) = \frac{as + bN\Omega}{s^2 + (N\Omega)^2} = \frac{as + bN\Omega}{(s - jN\Omega)(s + jN\Omega)}$$

then near $s = jN\Omega$,

$$K(s)G(s) \approx G(jN\Omega)\frac{(aj + b)N\Omega}{2jN\Omega}\frac{1}{s - jN\Omega}$$

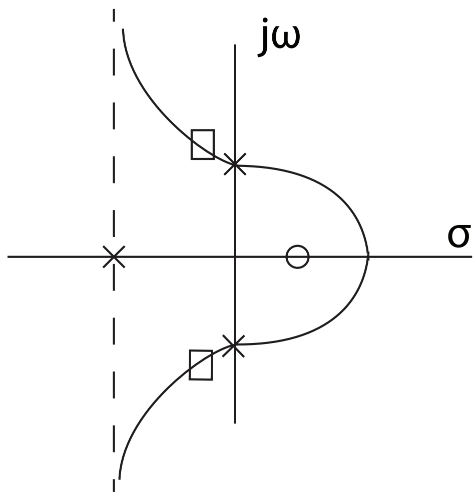
Using this approximation and setting $KG = -1$at $s = -\frac{1}{T} + jN\Omega$, we find

$$\begin{aligned} a =& \frac{2}{T}\mathrm{Re}(\frac{1}{G(jN\Omega)}) \\ b =& -\frac{2}{T}\mathrm{Im}(\frac{1}{G(jN\Omega)}) \end{aligned}$$

Example:

$$
\begin{aligned}
G(s) =& \frac{1}{s+1} \\
\Omega =& 1,\ N = 1,\ T = 10 \\
\Rightarrow a =& 0.2,\ b = -0.2 \\
\Rightarrow K(s) =& 0.2\ \frac{s-1}{s^2+1}
\end{aligned}
$$

Root locus:



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