Lift and Drag Primer

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1 Introduction

1.1 Objective

The objective of these notes is to provide a basic framework for the estimation of lift and drag on aircraft, with a particular focus on the dominant aircraft shape of a slender wing attached to a cylindrical fuselage at or near cruise conditions. The approach we describe is fairly standard across the aerospace industry though the details (and the notation) will likely be different. Also, while aerodynamic theory underpins much of the framework, we largely focus on the final results and simply give the topic names which should be investigated for a more complete understanding.

1.2 Definition of Lift and Drag

The **drag** is the aerodynamic force acting on the aircraft in the freestream direction. The **lift** is the aerodynamic force acting on the aircraft perpendicular to the freestream direction. While the perpendicular direction to the freestream is ambiguous (due to rotation about freestream direction), aerodynamic bodies are usually symmetric; by convention, the lift acts in the direction perpendicular to the freestream direction in the plane of symmetry.

1.3 Generation of Aerodynamic Forces

While we will discuss different sources of aerodynamic forces (in particular for drag), aerodynamic forces can only be due to the action of pressure and shear stresses on the aircraft. Thus, while the estimation of drag (in the framework discussed in these notes) will include contributions due to skin friction drag, wave drag, induced drag, and pressure drag, the ultimate manner in which these different contributions generated is through pressure and viscous stresses acting on the surface of the aircraft. Specifically, the aerodynamic force acting on the aicraft is given by the following integral,

$$\vec{F} = \int_{S_{wet}} \vec{\tau}\, dS - \int_{S_{wet}} p\vec{n}\, dS, \tag{1}$$

where S_{wet} is known as the **wetted surface area** and is the surface of the aircraft exposed to the air. $\vec{\tau}$ is the viscous stress vector acting on the aircraft surface. p is the static pressure and $\vec{n}$ is the outward-pointing surface normal. Thus, the first and second integrals represent the viscous stress and pressure forces, respectively.

1.4 Sources of Drag

As described above, we will break drag into different sources. These are defined as follows:

Skin friction drag: drag due to the action of (viscous) shear stresses on the aircraft surface.

Wave drag: drag created by pressure forces on the aircraft surface due to the presence of supersonic flow and, in particular, shock waves.

Pressure drag: drag created by pressure forces due to the presence of viscous boundary layers on the aircraft surface. As the boundary layers thicken, and in the extreme case when the boundary layers separate, the pressure drag increases. Since this drag is most significant when the flow is separated, aerodynamicists often refer to it as drag due to separation.

Induced drag: drag created by pressure forces due to the trailing vortex system (wake) arising from the generation of lift. This type of drag only exists in three-dimensional flows; thus, airfoils only have skin friction, wave, and pressure drag.

2 Airfoils

2.1 Lift and Drag Coefficients

The lift and drag forces on an airfoil are denoted L' and D' respectively. In aerodynamic analysis, common practice is to use non-dimensional coefficients when quantifying the lift and drag (and many other quantities). In two-dimensions, the lift and drag coefficients are defined as:

$$c_l \equiv \frac{L'}{q_\infty c}, \tag{2}$$

$$c_d \equiv \frac{D'}{q_\infty c}, \tag{3}$$

where q_∞ is the dynamic pressure in the freestream,

$$q_\infty \equiv \frac{1}{2}\rho_\infty V_\infty^2, \tag{4}$$

and ρ_∞ and V_∞ are the freestream density and speed, respectively. The chord of the airfoil, c, is used as the reference length scale.

Question: Why is the chord used as the reference length scale in defining the lift and drag coefficients? Note: the choice of reference length scale *is* arbitrary. Also, the choice of q_∞ is arbitrary. However, good reasons exist for both choices. What are these good reasons?

2.2 Skin Friction Drag

The estimation of skin friction drag is based on theoretical and experimental results for the skin friction acting on a flat plate. The skin friction drag on an airfoil is given by,

$$D'_f = \int_{s_{wet}} \vec{\tau} \cdot \vec{e}_\infty \, ds,$$

which is the first integral in Equation (1) reduced to two-dimensions and taken in the freestream direction (denoted by the unit vector $\vec{e}_\infty$). Note, a lowercase s is used to denote a line contour in two-dimensions as opposed to a surface in three-dimensions.

The dominant component of the viscous stress vector is the **wall shear stress**, τ_w, which is the viscous shear stress that acts tangent to the airfoil surface (we assume that the tangent vector points from the leading towards the trailing edge). Thus, the skin friction drag can be well-approximated by,

$$D'_f \approx \int_{s_{wet}} \tau_w \vec{t} \cdot \vec{e}_\infty \, ds,$$

where $\vec{t}$ is the airfoil surface tangent vector. Furthermore, since airfoils are streamlined bodies which are usually oriented close to the freestream direction, the surface tangent and the freestream directions will be nearly tangent, i.e. $\vec{t} \cdot \vec{e}_\infty \approx 1$. Thus, we can further approximate the skin friction drag as,

$$D'_f \approx \int_{s_{wet}} \tau_w \, ds.$$

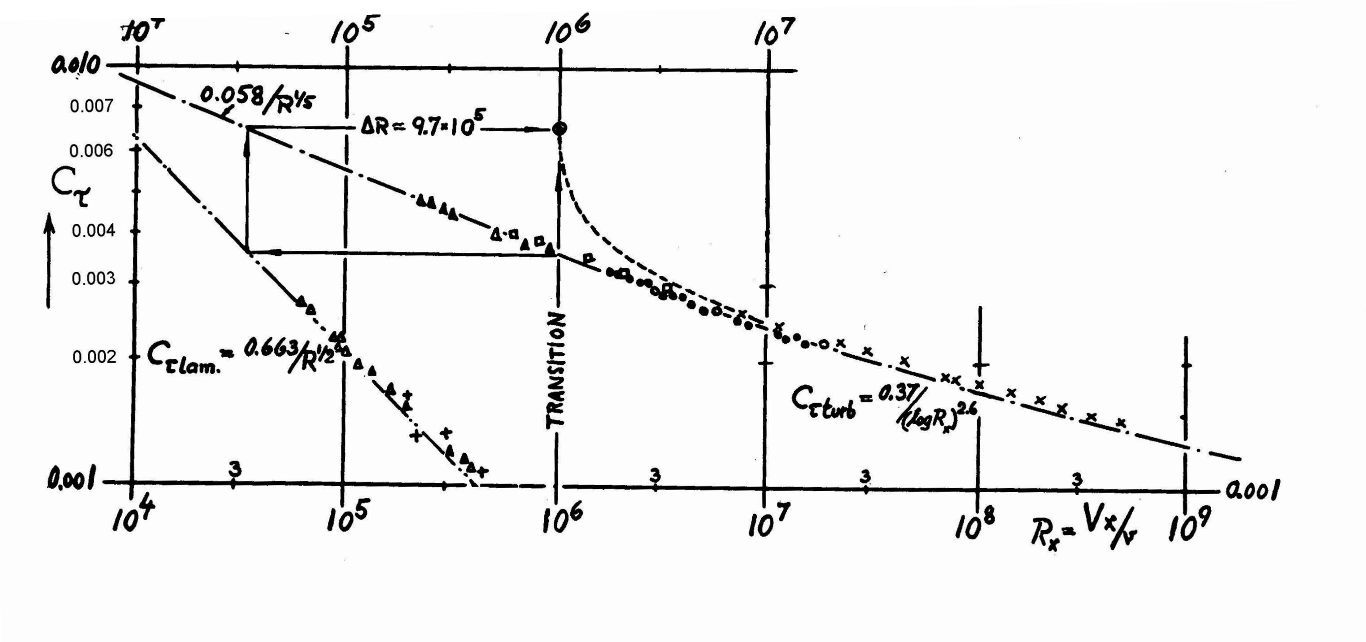

Figure 1: Skin friction coefficient, c_f, on a flat plate versus Re_x (from Hoerner [2]. Note: c_f is c_τ in figure).

As with the drag, we also non-dimensionalize the wall shear stress. Specifically, we define the **skin friction coefficient** as,

$$c_f \equiv \frac{\tau_w}{q_\infty}. \tag{5}$$

Substituting c_f into the friction drag gives,

$$D'_f \approx q_\infty \int_{s_{wet}} c_f \, ds. \tag{6}$$

At subsonic-to-transonsic Mach numbers, the skin friction coefficient on a smooth flat plate is known to be largely a function of only the Reynolds number based on distance from the leading edge, Re_x, defined as,

$$\mathrm{Re}_x \equiv \frac{V_\infty x}{\nu_\infty}, \tag{7}$$

where x is the distance from the leading edge and ν_∞ is the freestream kinematic viscosity. A plot of c_f versus Re_x is shown in Figure 1. Since airfoils are fairly streamlined bodies, the skin friction tends to behave similarly on flat plates and airfoils. As can be seen from the plot, two distinct regions exist. At low Re_x, the skin friction coefficient decreases proportional to $\mathrm{Re}_x^{-1/2}$. However, between $10^5 \leq \mathrm{Re}_x \leq 10^6$, the skin friction coefficient has two overlapping behaviors, the previous low skin friction data and new higher skin friction data. Then, for $\mathrm{Re}_x > 10^6$, the higher skin friction coefficient data decreases but at a slower rate than the low Reynolds number behavior.

The low Re_x behavior is characteristic of flows in which the boundary layer on the flat plate is laminar, while the high Re_x behavior is characteristic of turbulent boundary layer flows. The region between these limits from $10^5 \leq \mathrm{Re}_x \leq 10^6$ is known as the transition region. In this regime, the flow could be either laminar or turbulent depending on the specific geometry, conditions, etc. For an aircraft in cruise, a laminar boundary layer is steady, however, turbulent boundary layers have many small scale, unsteady fluctuations. These fluctuations are responsible for the increased skin friction coefficient relative to laminar boundary flows.

For a flat plate, the c_f curve in Figure 1 can be integrated with respect to x to produce the skin friction drag using Equation (6). However, careful integration for airfoils is not useful since several approximations already have been made in deriving Equation (6).

For many aerodynamic applications, the Reynolds numbers are so large that the flow is laminar for only a short distance downstream of the leading edge. Thus, in many practical situations, we can make reasonable estimates by assuming a representative, constant c_f based on the turbulent flat plate skin friction data. In this case, the skin friction drag is approximated by,

$$D'_f \approx q_\infty \overline{c_f} s_{wet}, \tag{8}$$

where $\overline{c_f}$ is a representative average value of c_f on the airfoil surface. This can be non-dimensionalized to give the skin friction drag coefficient,

$$c_{d\,f} \equiv \frac{D'_f}{q_\infty c} \approx \overline{c_f} \frac{s_{wet}}{c}. \tag{9}$$

Finally, since airfoils are thin, the wetted surface area is about twice the chord (i.e. upper + lower surfaces), thus,

$$c_{d\,f} \approx 2\overline{c_f}. \tag{10}$$

Note: the estimate for $\overline{c_f}$ can be performed using flat plate data when the airfoil is reasonably thin (say under 20%). For the Reynolds number range often of interest (approximately $10^7 < \mathrm{Re}_x < 10^9$), the data in Figure 1 would suggest that a reasonable value of $\overline{c_f}$ would be in the range 0.0015 to 0.002.

2.3 Pressure Drag

The pressure drag on a streamline body near design conditions is usually quite small. This can be observed from an estimate from Hoerner [2], Chapter 6, for the pressure drag at the minimum drag (or near design) conditions. Specifically, Hoerner states,

$$(c_{dp})_{\min} \approx 60 \left(\frac{t_{\max}}{c}\right)^4 c_{d\,f}.$$

Thus, the pressure drag at the minimum drag conditions scales with the friction drag and the thickness to the fourth power. For typical airfoils in commercial applications, the largest values of $t_{\max}/c \approx 0.2$ which leads to $(c_{dp})_{\min} \approx 0.1 c_{d\,f}$. Thus, even in the extreme case, the pressure drag is at most 10% of the friction drag near design conditions.

Unfortunately, for off-design conditions (i.e. angles of attack significantly different than the design condition), pressure drag is often the dominant source of drag and is very difficult to estimate except in extreme conditions. However, knowledge of these extreme conditions can be useful in setting upper limits on the drag. These upper limits can be compared with other pressure drag estimates, experimental and/or computational results as a 'sanity check'. For example, a flat plate oriented perpendicular to the upstream flow has a drag which is almost entirely pressure drag and is,

$$(c_{dp})_{normal} \approx 2.$$

This is a very large drag coefficient. **Question:** What is the average difference in static pressure between the two sides of the flat plate in order to generate this drag? Do you think the static pressure on the backside of the plate is less than, about equal to, or greater than the freestream static pressure?

Though the case of a cylinder is not discussed in these notes, the behavior of the drag on a cylinder is also very important in aerodynamics. Any one of the standard texts on aerodynamics will have a good discussion of drag on a cylinder (and/or sphere). See for example Anderson [1].

For moderate angles of attack, the drag increase above the minimum drag is often found to be proportional to ${c_l}^2$. Hoerner suggests the following as a rough estimate for the additional pressure drag above $(c_{dp})_{\min}$,

$$c_{dp} - (c_{dp})_{\min} \approx \left(c_{d\,f} + c_{dp}\right)_{\min} {c_l}^2. \tag{11}$$

2.4 Wave Drag

Wave drag occurs when the flow around the airfoil becomes supersonic in some region. Note, although the aircraft may be flying at subsonic speeds, the flow in the vicinity of the airfoil is often accelerated to speeds which are greater than the local speed of sound. For example, commercial transport aircraft fly at Mach numbers around 0.80 to 0.90; however, all of these aircraft have local supersonic regions over their wings. These supersonic regions eventually decelerate to lower speed (higher pressure conditions) and this deceleration occurs through shock waves. As we will see later in the course, supersonic flow (even without shock waves) can lead to drag, however, shock waves are the significant source of what is referred to as wave drag.

Unlike friction drag for which simple estimates can be made, wave drag is much harder to estimate. The following approach to estimate wave drag at higher subsonic (transonic) Mach numbers is based on Hoerner's book on drag [2], see Chapter 15. Hoerner suggests that the wave drag behaves as,

$$c_{dw} = K\left(\frac{M_\infty - M_{DD}}{1 - M_{DD}}\right)^3, \qquad (12)$$

where K is a constant which ranges from 0.35 for thinner airfoils (about 4% thick) to 0.50 for thicker airfoils (about 10% thick). M_{DD} is the drag divergence Mach number and is (loosely) defined as the freestream Mach number at which the wave drag rapidly increases. The rationale behind the cubic dependence of the wave drag on Mach number increases is tied to the theoretical result that the entropy change across a normal shock scales with $(M_n - 1)^3$ where M_n is the Mach number upstream of the shock; since the entropy change is related to the drag, the wave drag is expected to behave in a similar manner on airfoils. For $M_\infty < M_{DD}$, the wave drag is assumed to be zero.

Unfortunately, Hoerner does not offer any guidance on estimating M_{DD} and even states that the quality of the wave drag estimate is strongly dependent on the choice of M_{DD}. The best guidance I can offer is the following:

- Most airfoils begin to show substantial wave drag increases by about $M_\infty = 0.8$. This would be a reasonable estimate if no other information is available.
- Often past experience can help; set the drag divergence Mach number based on known results for similar airfoils.
- Compressible Computational Fluid Dynamic (CFD) simulations can be readily performed to estimate the wave drag with good accuracy.

2.5 Lift

The lift coefficient for an airfoil is well-approximated by thin-airfoil theory as,

$$c_l = \frac{2\pi}{\sqrt{1 - M_\infty^2}}\left(\alpha_\infty - \alpha_{l0}\right), \qquad (13)$$

where α_{l0} is the angle of attack at which the airfoil has zero lift. For symmetric (i.e. uncambered) airfoils, $\alpha_{l0} = 0$. For airfoils with camber, α_{l0} must be estimated from the camberline geometry using thin-airfoil theory. The compressibility correction factor, $\sqrt{1 - M_\infty^2}$, increases the lift coefficient with increasing Mach number. This scaling factor is known as the Prandtl-Glauert scaling factor.

3 Wings

3.1 Lift and Drag Coefficients

In three-dimensions, the lift and drag coefficients are defined as:

$$C_L \equiv \frac{L}{q_\infty S_{ref}}, \qquad (14)$$

$$C_D \equiv \frac{D}{q_\infty S_{ref}}. \qquad (15)$$

The reference area is arbitrary, however, the common choice for wings is the **planform area** (i.e. the area of the wing viewed from above).

3.2 Skin Friction Drag

The treatment of skin friction in three-dimensional wings is a straightforward extension of the estimation of skin fraction drag on two-dimensional airfoils. Replicating the derivation of Equation (6) in three-dimensions gives,

$$D_f \approx q_\infty \int_{S_{wet}} c_f \, dS. \tag{16}$$

As before, assuming an average c_f value, we may estimate the

$$C_{Df} \approx \overline{c_f} \frac{S_{wet}}{S_{ref}}. \tag{17}$$

For thin wings, $S_{wet} \approx 2S_{ref}$ when S_{ref} is the planform area. In this case, we arrive at $C_{Df} \approx 2\overline{c_f}$ which is identical to the airfoil result. As with the airfoil estimate, flat plate results can be used to approximate $\overline{c_f}$.

3.3 Pressure Drag

In three-dimensions, the estimation of pressure drag is even more difficult than in two-dimensions. The best approach would rely on a combination of experiments, flight test, and CFD. For quick estimation purposes, one might try simply using a three-dimensional version of the previous two-dimensional pressure drag estimates constructed by span-averaged properties where needed. For example,

$$(C_{Dp})_{\min} \approx 60 \left(\frac{t_{\max}}{c}\right)^4 C_{Df}.$$

And similarly, for the pressure drag away from the nominal design condition,

$$C_{Dp} - (C_{Dp})_{\min} \approx \left(C_{Df} + C_{Dp}\right)_{\min} {C_L}^2.$$

3.4 Wave Drag

The estimation of wave drag on a wing can be performed by integrating the wave drag from several airfoil sections of the wing and applying the airfoil wave drag estimation in Section 2.4. The only significant effect which must be accounted for is the sweep of the wing. By sweeping the wing, the airfoil sections 'observe' a decreased Mach number normal to their leading-edge. As a result, the airfoil is effectively observing a lower Mach number. The result is that drag divergence is delayed to higher Mach numbers than for wing without sweep. An approximate correction for this effect is given by Hoerner as,

$$C_{Dw} = \sin^3 \Lambda (C_{Dw})_0, \tag{18}$$

where Λ is the wing leading edge sweep and $(C_{Dw})_0$ is the wave drag coefficient for the same wing but with a straight leading-edge.

3.5 Induced Drag

The induced drag can be reasonably estimated from Prandtl's lifting line when the wing sweep is relatively small. The result for the induced drag coefficient is,

$$C_{Di} = \frac{{C_L}^2}{\pi \mathcal{R} e}, \tag{19}$$

where $\mathcal{R}$ is the wing aspect ratio,

$$\mathcal{R} = \frac{b^2}{S_{ref}},$$

and b is the span of the wing. e is known as the **span efficiency factor**. The best e that can be achieved according to lifting line theory is $e = 1$. Note that for most wings, e is a function of the angle of attack (the one exception is elliptic planform wings for which $e = 1$ for all α_∞). For reasonably well-designed wings near their minimum drag condition, e is generally close to one with a reasonable estimate being $e = 0.9$.

The dimensional form of the induced drag estimate from lifting line is also very useful to know. Specifically, the induced drag from Equation (19) can be written,

$$D_i = \frac{L^2}{\pi q_\infty b^2 e}.$$

Some interesting conclusions from this are:

- The induced drag increases at lower speeds (i.e. lower q_∞).
- The induced drag decreases with increasing span.
- The area of the wing does not enter the final result.

4 Lift

The lift is reduced by the wing tip vortices (which is also the same mechanism that leads to induced drag). Lifting line gives the following corrections:

$$C_L = \frac{a}{\sqrt{1 - M_\infty^2}} \left(\alpha_\infty - \alpha_{L0}\right), \tag{20}$$

where a is the modified lift-slope given by lifting-line theory,

$$a = \frac{2\pi}{1 + 2/\mathcal{R}}.$$

Note, the zero-lift angle will generally not be the same in two- and three-dimensions. We denote the wing zero-lift angle as α_{L0}.

References

[1] J.D. Anderson. *Fundamentals of Aerodynamics.* McGraw Hill, 2001. 3rd ed.

[2] S.F. Hoerner. *Fluid Dynamic Drag.* Hoerner Fluid Dynamics, 1965. 2nd ed.