A plate of length L has the following piecewise-defined BL edge velocity distribution $u_e(x)$:

$$u_e = 5u_0 \frac{x}{L} \qquad 0 \le x \le L/5$$
$$u_e = u_0 \qquad L/5 \le x \le L$$

The Reynolds number is $u_0 L/\nu = 10^7$.

1a) Determine the laminar boundary layer development for this edge velocity distribution using Thwaites' method.

1b) Determine the transition location using:

- Michel's criterion
- Granville's correlation
- The e^9 envelope method

Compare the results of the three methods and comment on the applicability of each method to this particular flow. For the e^9 method, use the $\tilde{n}$ vs Re_θ relations handed out in class, and for simplicity, perform the instability-growth calculation with respect to Re_θ instead of x.

The velocity components in a 2-D periodically-driven unsteady turbulent flow, such as a sinusoidally pitching airfoil, can be written as

$$u = \overline{u} + \tilde{u} + u' \qquad v = \overline{v} + \tilde{v} + v'$$

where: $\overline{u}$, $\overline{v}$ are temporal mean values
$\tilde{u}$, $\tilde{v}$ are unsteady periodic terms of zero temporal mean
u', v' are aperiodic chaotic fluctuations of zero temporal mean

Also, the above variables are defined such that u', v' do not contain any frequencies equal to the pitching frequency.

The governing unsteady TSL equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

2a) Using the time average definition of some function $f(x, y, t)$,

$$\overline{f}(x,y) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} f(x,y,t)\, dt$$

derive the time-averaged TSL equations for the periodic unsteady flow. Identify the dependent variables, and the terms which must be modeled to close the system.

An alternative to time-averaging is the ensemble average, which is the average of many different occurences of the flow sampled at a given phase angle.

$$< f(x,y,t) > \; = \; \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} f(x,y,t_i)$$

where t_i is the i_{th} occurence. Generally, u' and v' are *ergodic*, meaning that

$$< u'v' > \; = \; \overline{u'v'}$$

2b) Derive the ensemble-averaged TSL equations for the periodic unsteady flow, and again identify the dependent variables and the terms which must be modeled.

2c) Which averaging approach would you recommend for periodically forced unsteady flows?