Unit 9

Effects of the Environment

Readings:

Rivello 3.6, 3.7

T & G Ch. 13 (as background), sec. 154 specifically

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Thus far we have discussed mechanical loading and the stresses and strains caused by that. We noted, however, that the environment can have an effect on the behavior of materials and structures. Let's first consider:

Temperature and Its Effects

2 basic effects:

- expansion / contraction
- change of material properties

Look, first, at the former:

Concept of Thermal Stresses and Strains

Materials and structures expand and contract as the temperature changes. Thus:

$$\varepsilon^T = \alpha\,(\Delta\, T)$$

thermal strain (ε^T)

coefficient of thermal expansion (C.T.E.) (α) units: $\left[\dfrac{1}{\text{degrees}}\right]$

temperature change (ΔT)

If these thermal expansions / contractions are resisted by some means, then "thermal stresses" can arise. ***However***, "thermal stresses" is a misnomer, they are really…

"stresses due to thermal effects" -- stresses are **always** "mechanical"

(we'll see this via an example)

--> Consider a 3-D generic material.

Then we can write:

$$\varepsilon_{ij}^{T} = \alpha_{ij}\,\Delta T \qquad (9-1)$$

$i, j = 1, 2, 3$ (as before)

α_{ij} = 2nd order tensor

The *total strain* of a material is the sum of the *mechanical strain* and the *thermal strain*.

$$\underbrace{\varepsilon_{ij}}_{\text{total}} = \underbrace{\varepsilon_{ij}^{M}}_{\text{mechanical}} + \underbrace{\varepsilon_{ij}^{T}}_{\text{thermal}} \qquad (9-2)$$

- ("actual") total strain (ε_{ij}): that which you actually measure; the physical deformation of the part
- thermal strain (ε_{ij}^{T}): directly caused by temperature differences
- mechanical strain (ε_{ij}^{M}): that part of the strain which is directly related to the stress

Relation of mechanical strain to stress is:

$$\varepsilon_{ij}^{M} = \underbrace{S_{ijkl}}_{\text{compliance}} \sigma_{kl}$$

Substituting this in the expression for total strain (equation 9-2) and using the expression for thermal strain (equation 9-1), we get:

$$\varepsilon_{ij} = S_{ijkl}\,\sigma_{kl} + \alpha_{ij}\,\Delta T$$

$$\Rightarrow S_{ijkl}\,\sigma_{kl} = \varepsilon_{ij} - \alpha_{ij}\,\Delta T$$

We can multiply both sides by the inverse of the compliance…that is merely the elasticities:

$$S_{ijkl}^{-1} = E_{ijkl}$$

$$\Rightarrow \quad \boxed{\sigma_{kl} = E_{ijkl}\, \varepsilon_{ij} - E_{ijkl}\, \alpha_{ij}\, \Delta T}$$

This is the same equation as we had before except we have the thermal terms:

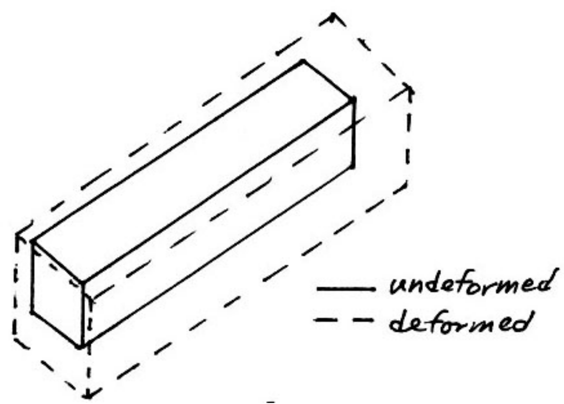
$$E_{ijkl}\, \alpha_{ij}\, \Delta\, T$$

--> so how does a "thermal stress" arise?

<u>Consider</u> <u>this</u> <u>example</u>:

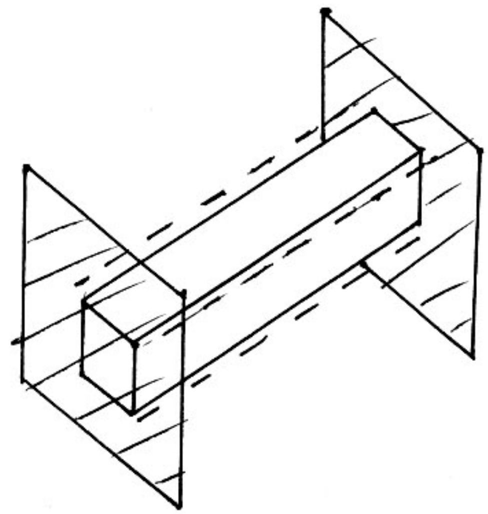
If you have a steel bar lying on a table and heat it, it will expand. Since it is unconstrained it expands freely and no stresses occur. That is, the thermal strain is equal to the total strain. Thus, the mechanical strain is zero and thus the "thermal stress" is zero.

Figure 9.1 **Free thermal expansion of a steel bar**



--> However, if the bar is constrained, say at both ends:

Figure 9.2 **Representation of constrained steel bar**



Then, as it is heated, the rod cannot lengthen. The thermal strain is the same as in the previous case <u>but</u> now the total strain is zero (i.e., no physical deformation).

Starting with (in one direction):

$$\varepsilon = \varepsilon^M + \varepsilon^T$$

with:

$$\varepsilon = 0$$

Thus, the mechanical strain is the negative of the thermal strain.

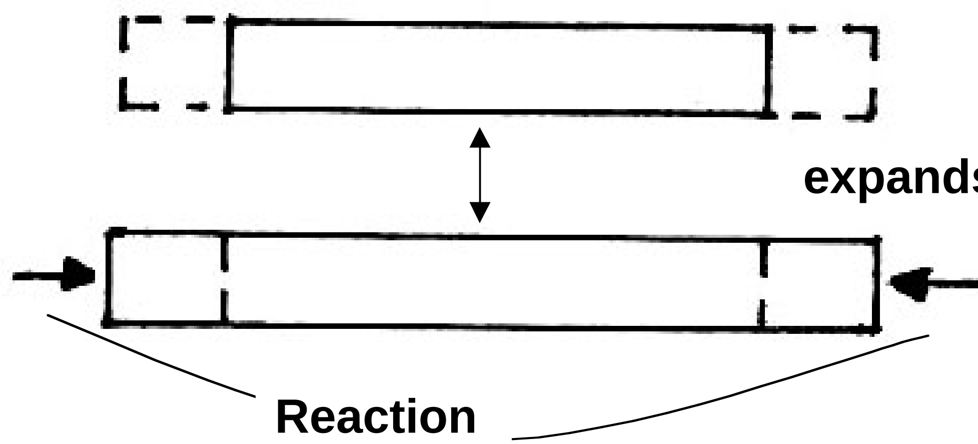
Stresses will arise due to the mechanical strain and these are the so-called “thermal stresses”.

Due to equilibrium there must be a reaction at the boundaries.

(must always have $\int \sigma dA$ = Force for equilibrium)

Think of this as a two-step process…

Figure 9.3 **Representation of stresses due to thermal expansion as two-step process**



$\varepsilon^M = -\varepsilon^T$

$\Rightarrow \varepsilon^{total} = 0$ (no physical deformation)

Values of C.T.E.'s

Note: $\alpha_{ij} = \alpha_{ji}$

- Anisotropic Materials

  6 possibilities: $\alpha_{11}, \alpha_{22}, \alpha_{33}, \alpha_{12}, \alpha_{13}, \alpha_{23}$

  $\Rightarrow \Delta T$ can cause shear strains

  → not true in "engineering" materials

- Orthotropic Materials

  3 possibilities: $\alpha_{11}, \alpha_{22}, \alpha_{33}$

  $\Rightarrow \Delta T$ only causes extensional strains

  Notes:
  1. Generally we deal with planar structures and are interested only in α_{11} and α_{22}
  2. If we deal with the material in other than the principal material axes, we can "have" an α_{12}

Transformation obeys same law as strain (it's a tensor).

2-D form:

$$\tilde{\alpha}_{\alpha\beta} = \ell_{\tilde{\alpha}\sigma}\ell_{\tilde{\beta}\gamma}\alpha_{\sigma\gamma}$$

$$\alpha^*_{11},\ \alpha^*_{22} \text{ (in-plane values)}$$

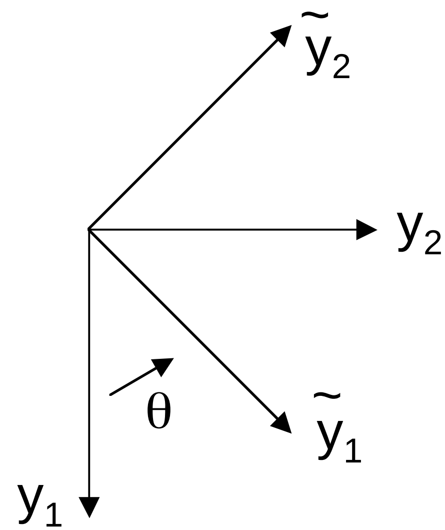
$$\alpha^*_{12} = 0 \text{ (in material axes)}$$

3-D form:

$$\alpha_{ij} = \ell_{\tilde{i}k}\ell_{\tilde{j}l}\alpha_{kl}$$

So, in describing deformation in some axis system at an angle θ to the principal material axes…..

Figure 9.4 **Representation of 2-D axis transformation**



θ + CCW

$$\tilde{\alpha}_{11} = \cos^2\theta\,\alpha_{11}^* + \sin^2\theta\,\alpha_{22}^*$$

$$\tilde{\alpha}_{22} = \sin^2\theta\,\alpha_{11}^* + \cos^2\theta\,\alpha_{22}^*$$

$$\tilde{\alpha}_{12} = \cos\theta\,\sin\theta\,\underbrace{\left(\alpha_{22}^* - \alpha_{11}^*\right)}$$

only exists if $\alpha_{11}^* \neq \alpha_{22}^*$
[isotropic ⇒ no shear]

- Isotropic Materials

  1 value: α is the same in all directions

Typical Values for Materials:

Material	**C.T.E.**
Steel	6
Aluminum	12.5
Titanium	5
Uni Gr/Ep (along fibers)	-0.2
Uni Gr/Ep (perpendicular to fibers)	16

Units: x 10^{-6}/°F
µin/in/°F
strain/°F

⇒ µstrain/°F

Notes:

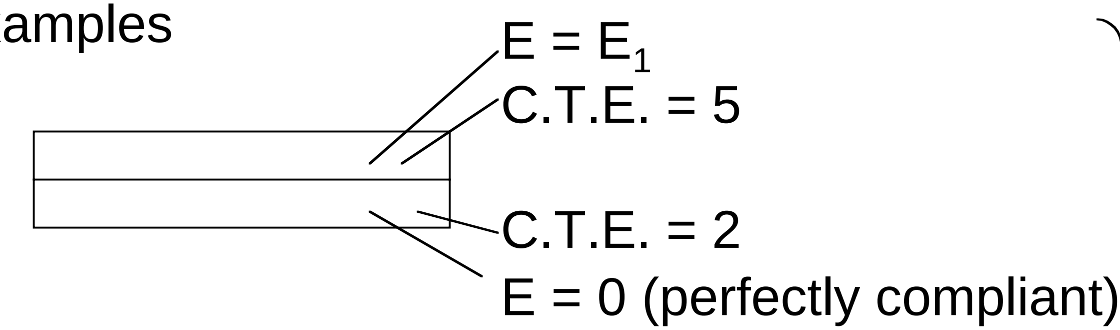
- Graphite/epoxy has a *negative* C.T.E. in the fiber direction so it contracts when heated

  Implication: by laying up plies with various orientation can achieve a structure with a C.T.E. equal to zero in a desired direction

- C.T.E. of a structure depends on C.T.E. and elastic constants of the parts

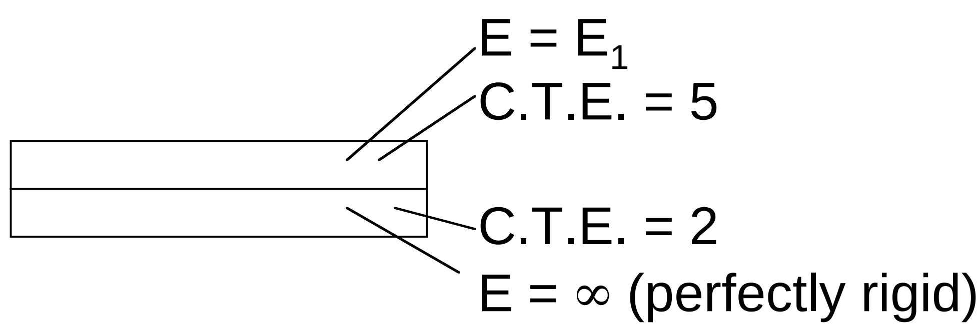
  2 examples

1)



total α of structure = 5

2)



total α of structure = 2

- $\alpha = \alpha(T) \Rightarrow$ C.T.E. is a function of temperature (see MIL HDBK 5 for metals). Can be large difference.

Implication: a zero C.T.E. structure may not truly be attainable since it may be C.T.E. at T_1 but not at T_2 !

--> Sources of temperature differential (heating)

- ambient environment (engine, polar environment, earth shadow, tropics, etc.)
- aerodynamic heating
- radiation (black-body)

--> Constant ΔT (with respect to spatial locations)

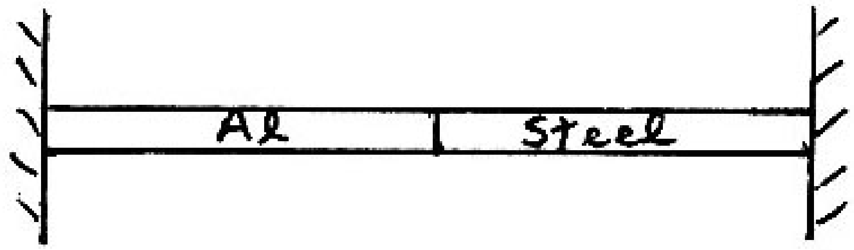
In many cases, we are interested in a case where ΔT (from some reference temperature) is constant through-the-thickness, etc.

- thin structures
- structures in ambient environment for long periods of time

Relatively easy problem to solve. Use:

- equations of elasticity
- equilibrium
- stress-strain

Example 1 - 2-material bar

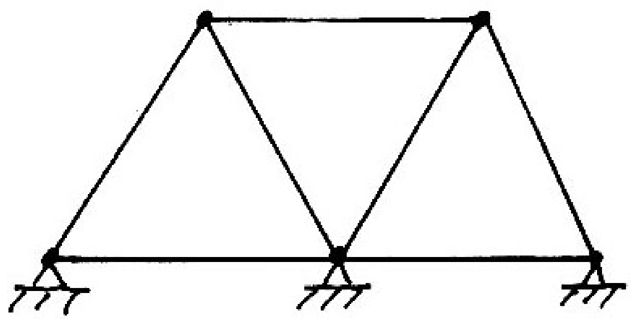


Total deformation is zero, but there will be nonzero total strain in the aluminum and steel

--> stress is constant throughout

--> match deformations

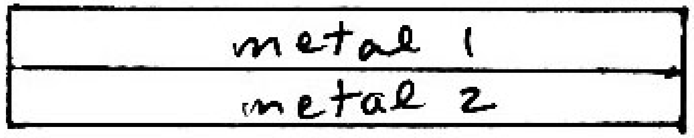
Example 2 - Truss



--> use equilibrium for forces in each member

--> match displacements at each node

Example 3 - bimetallic strip



Each metal has different α's.

What will happen?

Bending!

--> Concept of self-equilibrating stresses

Must always be in equilibrium. General equation is:

$$\int \sigma dA = F$$

where: F = externally applied force

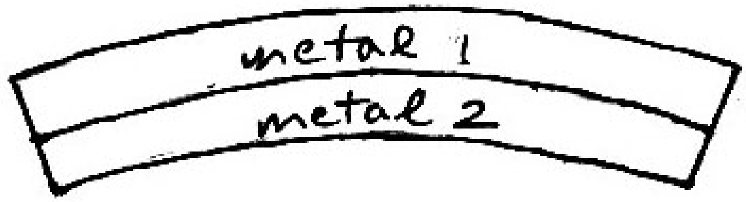
If F = 0, can we still have stresses?

Yes, but they must be "self-equilibrating" (satisfy equilibrium in and of themselves):

$$\int \sigma dA = 0$$

This is the case of free expansion (thermal) of structures with varying properties or spatially-varying ΔT (we'll address this in a bit)

If $\alpha_1 > \alpha_2$ and $\Delta T > 0$



$\varepsilon^T_{ij_1} > \varepsilon^T_{ij_2}$

(we'll see more about this bending later)

Bimetallic strip used as temperature sensors!

--> <u>ΔT varies spatially</u> (and possibly with time as well, we analyze at any constant in time)

Must determine ΔT by looking at heat flux into structure. Three basic methods:

- induction
- convection
- radiation

Convection most important in aircraft:

Aerodynamic Heating

look at adiabatic wall temperature

$$T_{AW} = \left[1 + \frac{\gamma - 1}{2} r M_\infty^2\right] T_\infty$$

where:

γ = specific heat ratio (1.4 for air)

r = "recovery factor" (0.8 - 0.9)

M_∞ = Mach number

T_∞ = ambient temperature (°K)

T_{AW} is maximum temperature obtained on surface (for zero heat flux)

Note: @40,000 ft. $M = 2 \Rightarrow T_{AW} = 230°F$

$M = 3 \Rightarrow T_{AW} = 600°F$

$\Rightarrow$ *(much above M = 2, cannot use aluminum since properties are too degraded)*

worse in reentry

Source of heat here is from air boundary layer:

$$q = h \quad (T_{AW} - T_s)$$

heat flux [watts/M^2]

heat transfer coefficient (convective constant)

surface temperature of body

(h is determined from boundary layer theory)

Radiation

Always important but especially in space (or at high temperature in atmosphere).

--> 2 considerations

1. Emissivity

--> surface emits heat

$$q = -\varepsilon\,\sigma\,T_s^4$$

heat flux

surface temperature

Stefan-Boltzmann constant

emissivity
(a material property)

2. <u>Absorptivity</u>

$$q = \alpha\,I_s\,\lambda$$

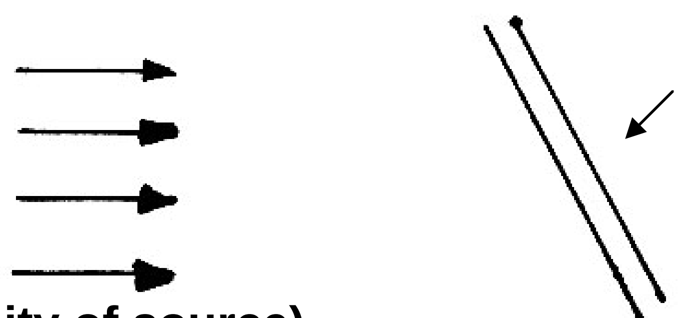
heat flux

angle factor

intensity of source

absorptivity
(a material property)

Figure 9.5 **Representation of heat flux impinging on structure**



Heat conduction

The general equation for heat conduction is:

$$q_i^T = -k_{ij}^T \frac{\partial T}{\partial x_j}$$

where:

T = temperature [°K]

q_i^T = heat flux per unit area in i direction $\left[\frac{\text{watts}}{\text{m}^2}\right]$

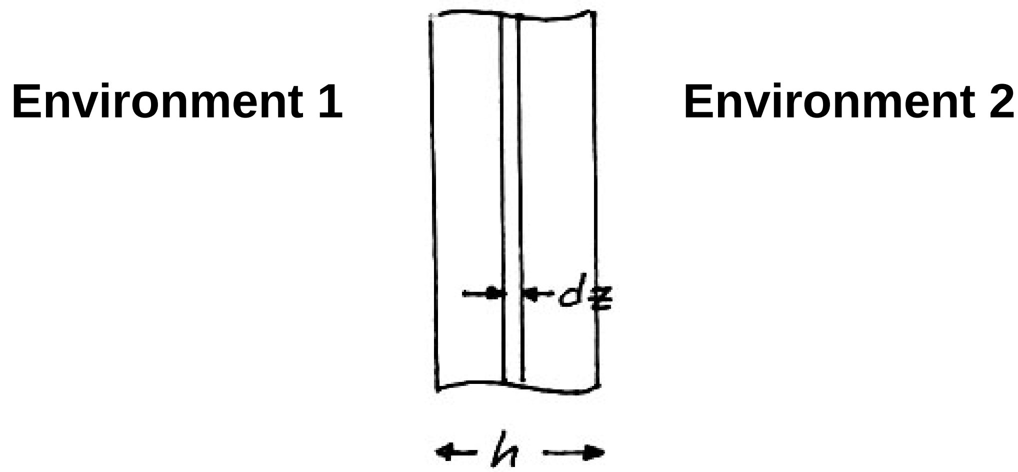
k_{ij}^T = thermal conductivity $\left[\frac{\text{watts}}{\text{m °K}}\right]$

(material properties)

The k_{ij}^T are second order tensors

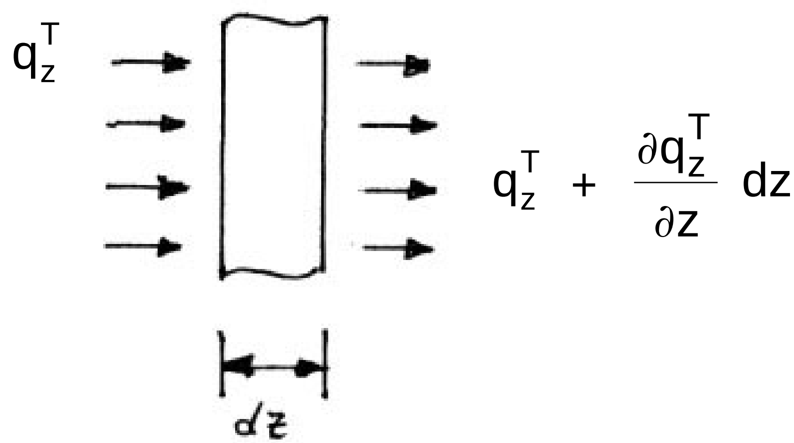
consider:

Figure 9.6 **Representation of structure exposed to two environments**



look at a strip of width dz:

Figure 9.7 **Representation of heat flow through infinitesimal strip of material**



Do a balance of energy:

$$q_z^T - \left(q_z^T + \frac{\partial q_z^T}{\partial z} \right) dz = \rho C \frac{\partial T}{\partial t} dz$$

heat flux on side 1 - heat flux on side 2 = change in stored heat in strip

where:

ρ = density
C = specific heat capacity
t = time

this becomes:

$$-\frac{\partial q_z^T}{\partial z} = \rho C \frac{\partial T}{\partial t}$$

Recalling that:

$$q_z^T = -k_z^T \frac{\partial T}{\partial z}$$

we have:

$$\frac{\partial}{\partial z}\left(k_z^T \frac{\partial T}{\partial z}\right) = \rho C \frac{\partial T}{\partial t}$$

If k_z^T and ρc are constant with respect to z (for one material they are), then we get:

$$\boxed{\frac{k_z^T}{\rho C} \frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t}}$$

Fourier's equation

We call:

$$\frac{k_z^T}{\rho C} = \text{thermal diffusivity}$$

More generally, for 3-D variation:

$$\frac{\partial}{\partial x}\left(k_x^T \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y^T \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z^T \frac{\partial T}{\partial z}\right) = C\rho \frac{\partial T}{\partial t}$$

thermal conductivities in x, y, and z directions

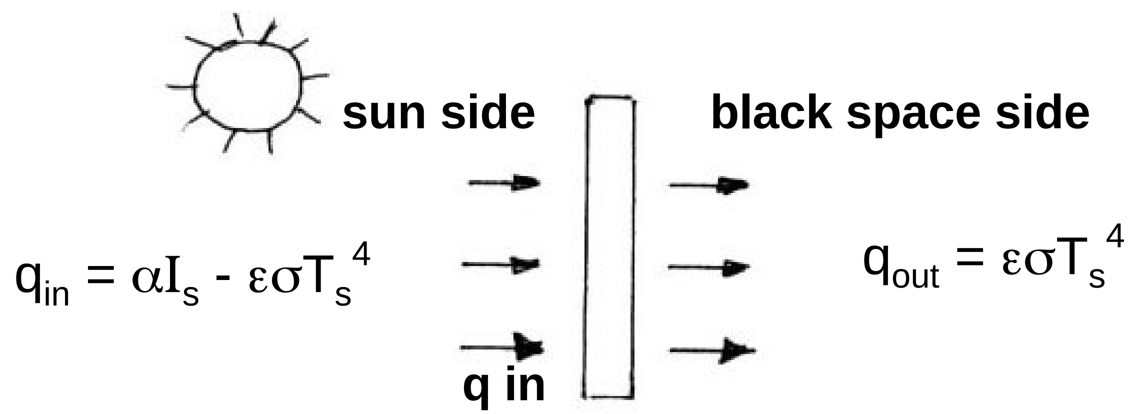
Bottom line: use these equations to solve for temperature distribution in structure subject to B.C.'s

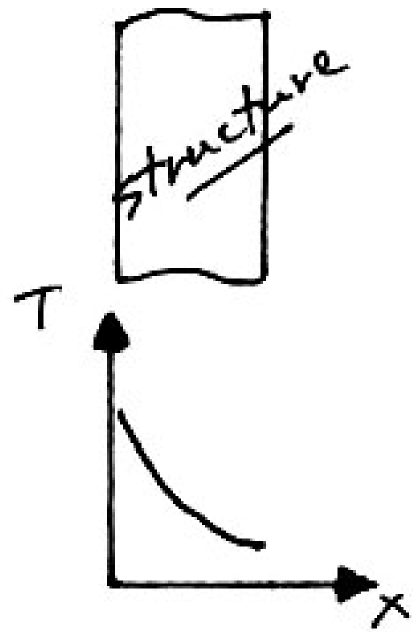
$T\ (x, y, z)$ gives $\Delta T\ (x, y, z)$

Note: These variations can be significant

Example:

Figure 9.8 **Representation of plate in space**





could get other cases where T peaks in the center, etc.

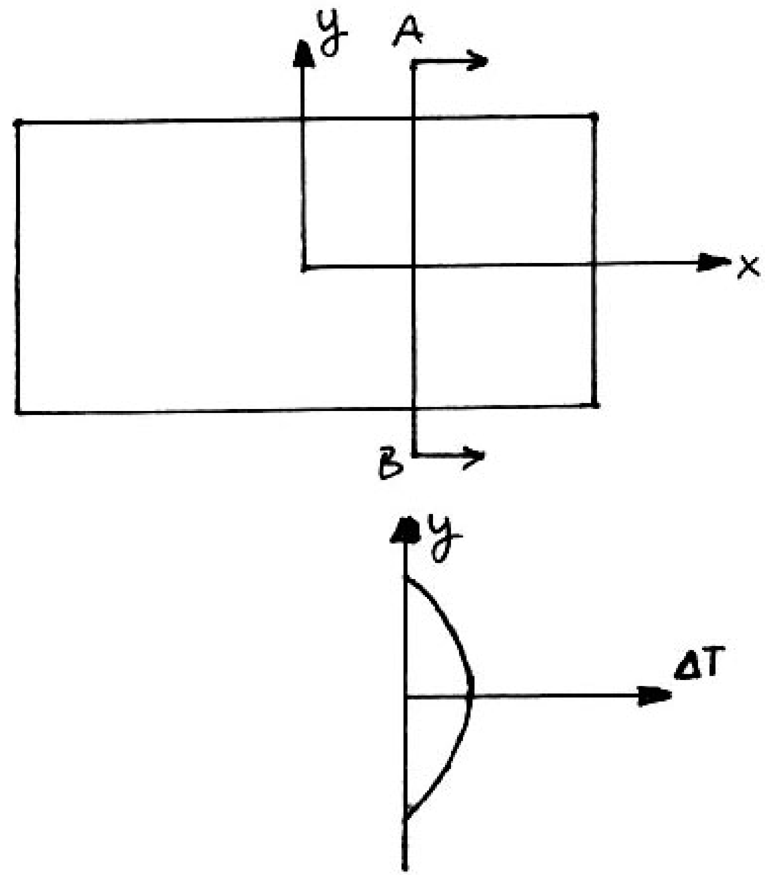
Result:

⇒ Internal stresses (generally) arise if T varies spatially. (unless it is a linear variation which is unlikely given the governing equations).

Why?

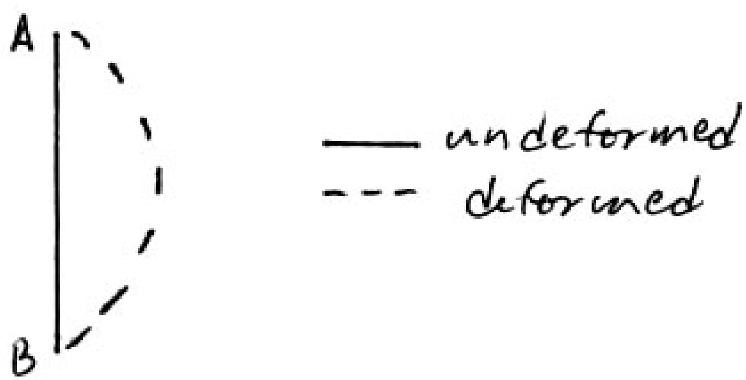
consider an isotropic plate with ΔT varying only in the y-direction

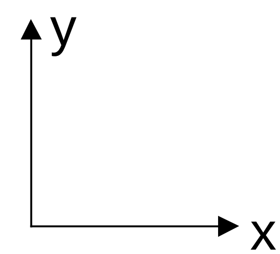
Figure 9.9 **Representation of isotropic plate with symmetric y-variation of ΔT about x-axis**



(for the time being, limit ΔT to be symmetric with respect to any of the axes)

At first it would seem we get a deformation of a typical cross-section A-B as:





This basic shape would not vary in x.

Note, however, that this deformation in the x-direction (u) varies in y.

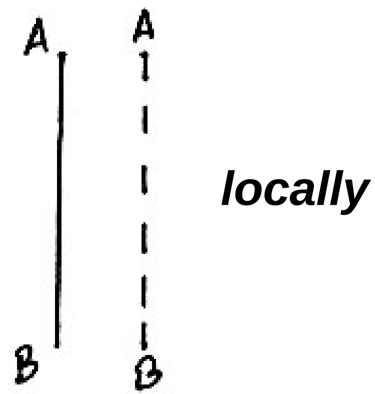
$$\Rightarrow \quad \frac{\partial u}{\partial y} \neq 0$$

$\Rightarrow$ shear strain exists!

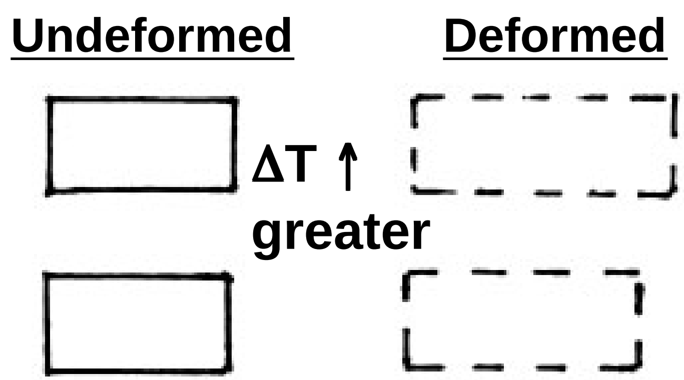
<u>**But**</u>, ΔT only causes <u>extensional</u> <u>strains</u>. Thus, this deformation cannot occur.

(in some sense, we have "plane sections must remain plane")

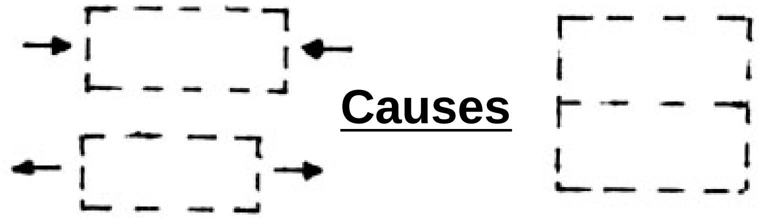
Thus, the deformation must be:



In order to attain this deformation, stresses must arise. Consider two elements side by side



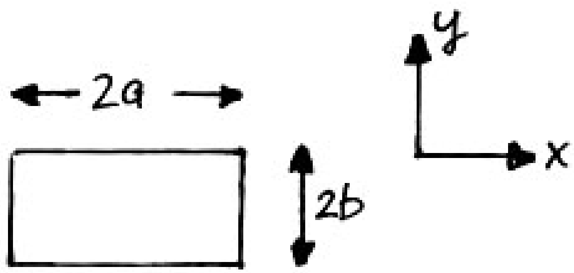
These two must deform the same longitudinally, so there must be stresses present to compress the top piece and elongate the bottom piece



Thus:

$$\varepsilon_x = \varepsilon_x\ (x)$$
$$\varepsilon_y = \varepsilon_y\ (y)$$

This physical argument shows we have thermal strains, mechanical strains and stresses.



self-equilibrating

$$\int_{-a}^{a} \sigma_y dx \;=\; 0$$

$$\int_{-b}^{b} \sigma_x dy \;=\; 0$$

Solution Technique

No different than any other elasticity problem. Use equations of elasticity subject to B. C.'s.

- exact solutions
- stress functions

recall:

$$\nabla^4 \phi = -E\alpha\nabla^2(\Delta T) - (1 - \nu)\nabla^2 V$$

- etc.

(see Timoshenko)

--> Does this change for orthotropic materials?

NO (stress-strain equations change)

We've considered *Thermal Strains and Stresses*, now let's look at the other effect:

Degradation of Material Properties (due to thermal effects)

Here there are two major categories

(see Rivello)

1. "Static" Properties

- Modulus, yield stress, ultimate stress, etc. change with temperature (generally, T$\uparrow$ $\Rightarrow$ property $\downarrow$)
- Fracture behavior (fracture toughness) goes through a transition at "glass transition temperature"

$$\text{ductile} \underset{T_g}{\rightarrow} \text{brittle}$$

Figure 9.10 **Representation of variation of ultimate stress with temperature**

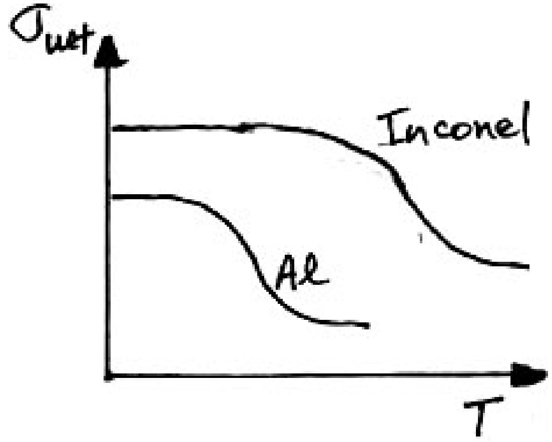
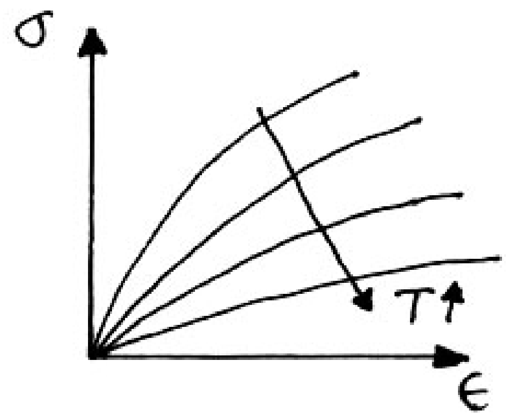


Figure 9.11 **Representation of change in stress-strain behavior with temperature**



(generally, behavior is more ductile as T increases)

--> Thus, must use properties at appropriate temperature in analysis

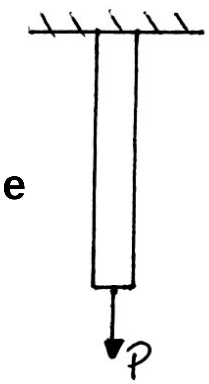
MIL-HDBK-5 has much data

2. “Time-Dependent” Properties

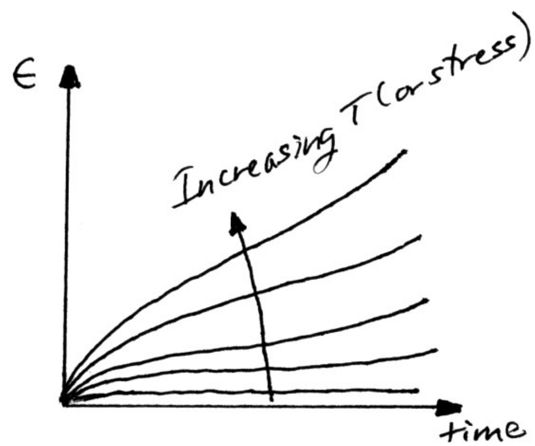
There is a phenomenon (time-dependent) in materials known as *creep*. This becomes especially important at elevated temperature.

Figure 9.12 **Representation of creep behavior**

Hang a load P and monitor strain with time



resulting strain-time behavior



This keeps aluminum from being used in supersonic aircraft in critical areas for aerodynamic heating.

"Other" Environmental Effects

- Temperature tends to be the dominating concern, but others may be important in both areas
  - atomic oxygen degrades properties
  - UV degrades properties
  - etc.
- Same effects may cause environmental strains like thermal strains:

Example - moisture

Materials can absorb moisture. Characterized by a "swelling coefficient" = β_{ij}

Same "operator" as α_{ij} (C. T. E.) except it operates on moisture concentration, c:

$$\underbrace{\varepsilon_{ij}^{s}}_{\text{"swelling" strain}} = \underbrace{\beta_{ij}}_{\text{swelling coefficient}} \underbrace{c}_{\text{moisture concentration}}$$

and then we have:

$$\underbrace{\varepsilon_{ij}}_{\text{total}} = \varepsilon_{ij}^{M} + \varepsilon_{ij}^{T} + \varepsilon_{ij}^{S}$$

This can be generalized such that the strain due to an environmental effect is:

environmental strain

$$\boxed{\varepsilon_{ij}^{E} = \chi_{ij}\, X}$$

environmental operator

environmental scalar

and the total strain is the sum of the mechanical strain(s) and the environmental strains

A strain of this "type" has become important in recent work. This deals with the field of

Piezoelectricity

A certain class of materials, known as piezoelectronics, have a coupling between electric field and strain such that:

electric field *causes* deformation/strain

strain *results in* electric field

This can be looked at conceptually the same way as environmental strains except electric field is a vector (not a scalar). Thus, the basic relationship is:

piezoelectric

$$\varepsilon_{ij}^{p} = d_{ijk} E_k$$

where:

E_k = electric field

d_{ijk} = piezoelectric constant

units = [strain/field]

a key difference here is that the "operator" (d_{ijk}) is a third-order tensor

(how transform? 3 direction cosines)

And we add this strain to the others to get the total strain

(consider the case with only mechanical and piezoelectric strain)

$$\varepsilon_{ij} = \varepsilon_{ij}^{M} + \varepsilon_{ij}^{p}$$

Again, only the mechanical strain is related *directly* to the stress:

$$\varepsilon_{ij} = S_{ijmn}\,\sigma_{mn} + d_{ijk}\,E_k$$

inverting gives:

$$\sigma_{ij} = E_{ijmn}\,\varepsilon_{mn} - E_{ijmn}\,d_{mnk}\,E_k$$

(watch the switching of indices!)

thus we have “piezoelectric-induced” stresses of:

$$E_{ijmn}\,d_{mnk}\,E_k$$

if the piezoelectric expansion is physically resisted.
Again, equilibrium $\left(\int \sigma = F\right)$ **must** be satisfied.

But, unlike environmental cases, the electric field is not just an external parameter from some uncoupled equation of state but there is a coupled equation:

$$D_i = e_{ik} E_k + d_{inm} \sigma_{mn}$$

note switch in indices since this is transpose of dielectric constant from previous equation

where:

e_{ik} = dielectric constant
D_i = electrical charge

--> "Normally", when piezoelectric materials are utilized, "E-field control" is assumed. That is, E_k is the independent variable and the electrical charge is allowed to "float" and take on whatever value results. But, when charge constraints are imposed the simultaneous set of equations:

$$\sigma_{mn} = E_{mnij} \varepsilon_{ij} - E_{mnij} d_{ijk} E_k$$

$$D_i = e_{ik} E_k + d_{inm} \sigma_{mn}$$

must be solved. This is coupled with any other sources of strain (mechanical, etc.)

Piezoelectrics useful for

- sensors
- <u>control</u> <u>of</u> <u>structures</u> (particularly dynamic effects)

<u>Note</u>: electrical folk use a <u>very</u> different notation (e.g., S = strain)

Now that we've looked at the general "causes" of stress and strain and how to manipulate, etc., consider general structures and stress and strain in that context.