

16.225 - Computational Mechanics of Materials
Homework assignment # 3
Handed out: 10/15/03
Due: 10/29/03

1. Behavior of the 3-node simplex element in the incompressible limit. Using the 3-node simplex element, repeat the plane-strain, plate-with-hole calculations of the previous assignment using traction boundary conditions for the case $-h 0.125$ and for the following values of the Poisson ratio: $\nu = 0.49, 0.499, 0.49999$. Comment on the results.
2. Behavior of the 3-node simplex element in problems with singularities. Consider the case of a semi-infinite crack coincident with the half axis $x_2 = 0, x_1 < 0$ subjected to mode I remote loading. Restrict the analysis to the rectangular domain $[-L, L] \times [0, L]$. The boundary conditions on the line $x_2 = 0$ are: traction-free for $x_1 < 0, u_2 = 0$ (symmetry) for $x_1 \geq 0$. On the remainder of the boundary the displacements are constrained to equal the K -field:

$$\begin{aligned}u_1 &= \frac{K}{2\mu} \sqrt{\frac{r}{2\pi}} \cos(\theta/2) [\kappa - 1 + 2 \sin^2(\theta/2)] \\u_2 &= \frac{K}{2\mu} \sqrt{\frac{r}{2\pi}} \sin(\theta/2) [\kappa + 1 - 2 \cos^2(\theta/2)]\end{aligned}\tag{1}$$

where $\kappa = 3 - 4\nu$, μ is the shear modulus, ν Poisson's ratio, r is the distance to the origin and the polar angle θ is measured from the x_1 -axis. Compute the finite element solution for meshes corresponding to

values of the mesh scaling factor $h = 1/2^p$ with $p = 0, \dots, 5$. Plot the deformed mesh and the contours of σ_{11}, σ_{22} . Also plot the energy-norm error $\| \mathbf{u}_h - \mathbf{u} \|_E$, computed by numerical quadrature, as a function of the number of elements E in *log – log* scale. Comment on the convergence properties of this finite element approximation.

3. Write a 6-node simplex element *elmt01* to do plane stress/strain linear elasticity. Verify that your element and, thus, the implementation, satisfies the patch test.
4. Repeat the calculations in [1](#). using the 6-node element. Comment on the results.
5. Repeat the calculations in [2](#). using the 6-node element. Comment on the results.