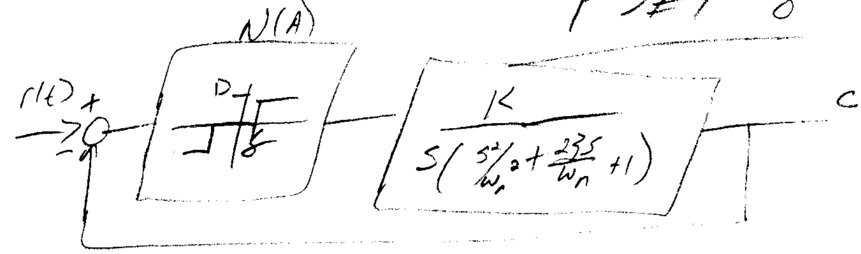
PSET 8

3.23



$$G(s)=\frac{K}{s\left(\frac{s^2}{\omega_n^2}+\frac{2\zeta s}{\omega_n}+1\right)}$$

Limit Cycle when $1+N(A)G(s)=0$

or $N(A)G(s)=-1$

Conditions ① $|N(A)||G(s)|=1$ ② $\angle N(A)+\angle G(s)=-180$

Cond ② done first to find ω

② $\angle N(A)$ is 0

$$0-90-\tan^{-1}\left(\frac{2\zeta\omega_n}{1-\omega^2/\omega_n^2}\right)=0$$

$$\tan^{-1}\left(\frac{2\zeta\omega_n}{1-\omega^2/\omega_n^2}\right)=90$$

$\tan^{-1}$ is undefined at 90 meaning $1-\frac{\omega^2}{\omega_n^2}=0$

or $\underline{\omega=\omega_n}$.

① $$|G(j\omega)|\Big|_{\omega=\omega_n}=\frac{K}{2\zeta\omega_n}$$

$$|N(A)|=\frac{4D}{\pi A}\sqrt{1-\left(\frac{\delta}{A}\right)^2}$$

$$\left(\frac{K}{2\zeta\omega_n}\right)\left(\frac{4D}{\pi A}\right)\sqrt{1-\left(\frac{\delta}{A}\right)^2}=1$$

$$\left(\frac{2KD}{\zeta\omega_n\pi}\right)^2\left(\frac{1}{A^2}\right)-\left(\frac{2KD\delta}{\zeta\omega_n\pi}\right)^2\left(\frac{1}{A^4}\right)=1$$

$$A^4-\left(\frac{2KD}{\zeta\omega_n\pi}\right)^2A^2+\left(\frac{2KD\delta}{\zeta\omega_n\pi}\right)^2=0$$

$$A=\sqrt{\frac{\left(\frac{2KD}{\zeta\omega_n\pi}\right)^2}{2}\pm\frac{1}{2}\sqrt{\left(\frac{2KD}{\zeta\omega_n\pi}\right)^4-4\left(\frac{2KD\delta}{\zeta\omega_n\pi}\right)^2}}$$

$$A=\frac{2KD}{\zeta\omega_n\pi}\sqrt{\frac{1}{2}\pm\frac{1}{2}\sqrt{1-\frac{4\delta^2}{\left(\frac{2KD}{\zeta\omega_n\pi}\right)^2}}}$$

$$A=\frac{\sqrt{2}\,KD}{\zeta\omega_n\pi}\sqrt{1\pm\sqrt{\underbrace{1-\left(\frac{\delta\omega_n\pi\zeta}{KD}\right)^2}_{>0}}}$$

A must be real, so >0

$1>\frac{\delta\omega_n\pi\zeta}{KD}\Rightarrow K>\frac{\delta\omega_n\pi\zeta}{D}$ and above 0

$$\boxed{0<K<\frac{\zeta\delta\omega_n\pi}{D}}$$

→ If $K=\frac{2\delta\omega_n\pi\zeta}{D}$

conditions still hold, so $\omega=\omega_n$

$$\frac{4D}{\pi A}\sqrt{1-\left(\frac{\delta}{A}\right)^2}\left(\frac{2\delta\omega_n\pi\zeta}{D\,2\zeta\omega_n}\right)=1$$

$$\frac{16\delta^2}{A^2}\left(1-\left(\frac{\delta}{A}\right)^2\right)=1$$

$$16A^2\delta^2-16\delta^4=A^4$$

$$A^4-16A^2\delta^2+16\delta^4=0$$

$$A=\sqrt{\frac{16\delta^2}{2}\pm\frac{1}{2}\sqrt{16^2\delta^4-4\cdot16\delta^4}}$$

$$A=\sqrt{8\delta^2\pm\frac{4\delta^2}{2}\sqrt{12}}$$

$$A=\sqrt{8\delta^2\pm4\delta^2\sqrt{3}}$$

$$A=2\delta\sqrt{2\pm\sqrt{3}}$$

$\boxed{A=3.9\delta}$, 1.04δ

For A slightly greater than 1.04δ $N(A)\uparrow$ so this is unstable

For $A>3.9\delta$ $N(A)\downarrow$ so stable