Handout 8: Lead compensation

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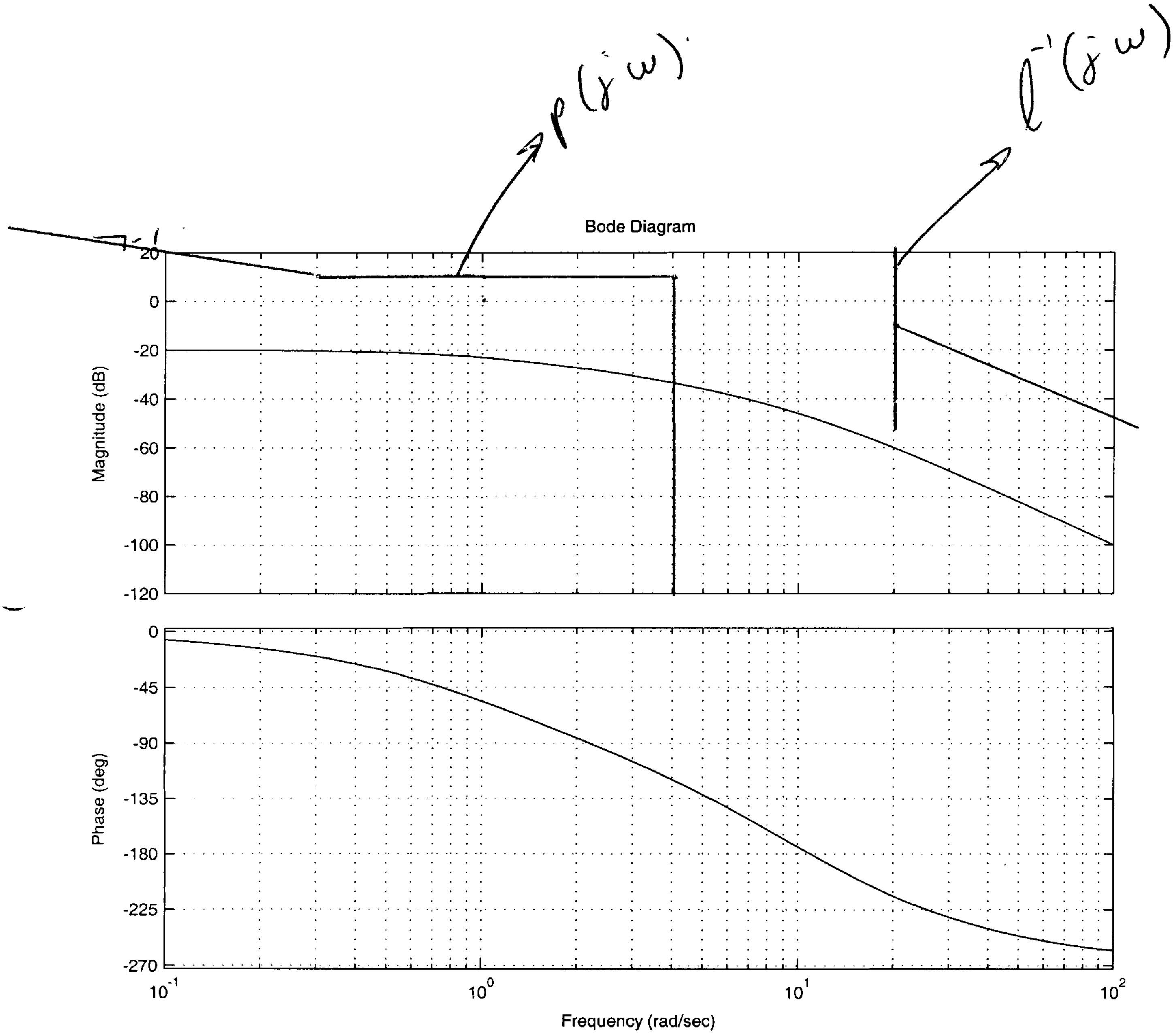
March 3, 2004

Lead Compensation goals: Raise phase (and gain) at high frequencies while not touching low-frequency system's characteristics: Can extend *bandwidth* of system.

Canonical lead element:

$$K_{lead}(s) = \frac{s/a+1}{s/b+1}, \quad 0 \leq a < b.$$

Typical lead Bode Plot:



p(jω)
l⁻¹(jω)
Bode Diagram
Magnitude (dB)
20
0
-20
-40
-60
-80
-100
-120
Phase (deg)
0
-45
-90
-135
-180
-225
-270
10^-1
10^0
10^1
10^2
Frequency (rad/sec)

$K_p = 110$

Using Lead-Lag / PID compensation

Plant under study:

$$G(s) = \frac{1/10}{(s+1)(s/10+1)^2}$$

' Requirements: Want to have good tracking ($p(j\omega)$), insensitivity to high frequency unmodelled dynamics ($l(j\omega)$, decent PM.

Compensation Scheme: We first adjust the gain K in the feedback loop to 110.

Phase Margin is

Gain Margin is

BW is

Lead compensation:

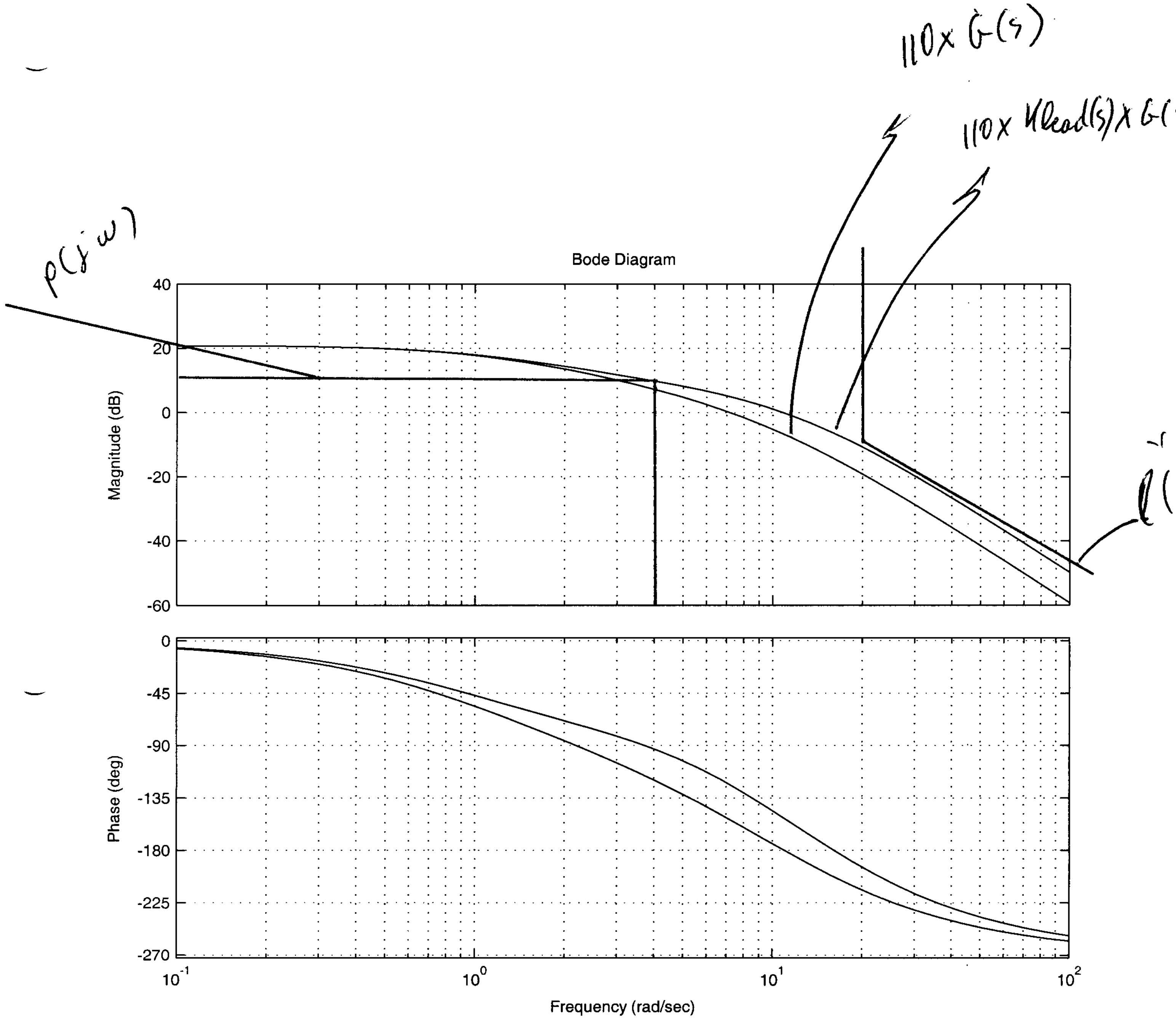
$$K_{lead}(s) = \frac{s/c + 1}{s/d + 1}$$

Lag compensation:

$$K_{lag}(s) = \frac{s + a}{s + b}$$

Final design: Bode plot

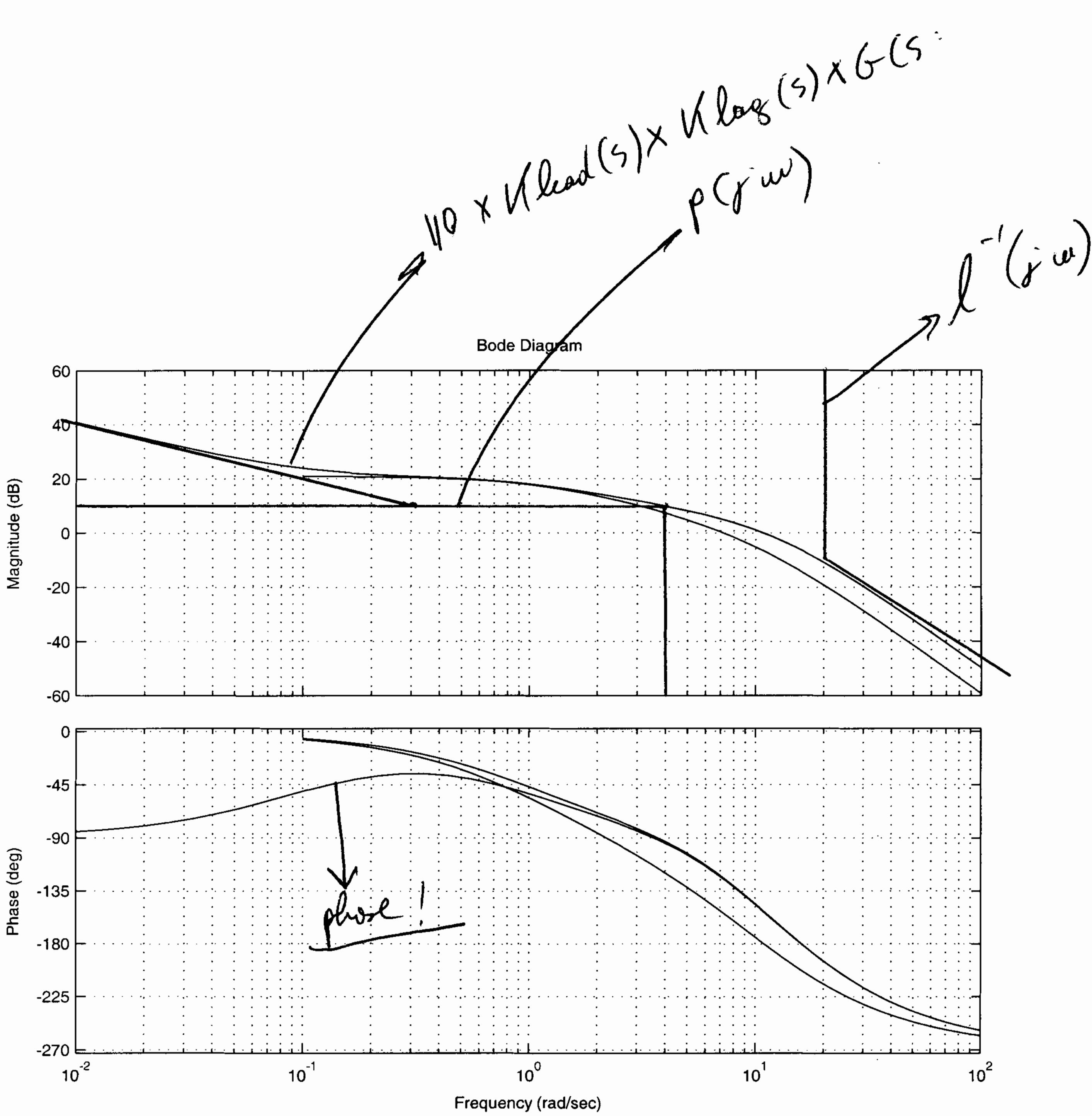
Final design: Root locus

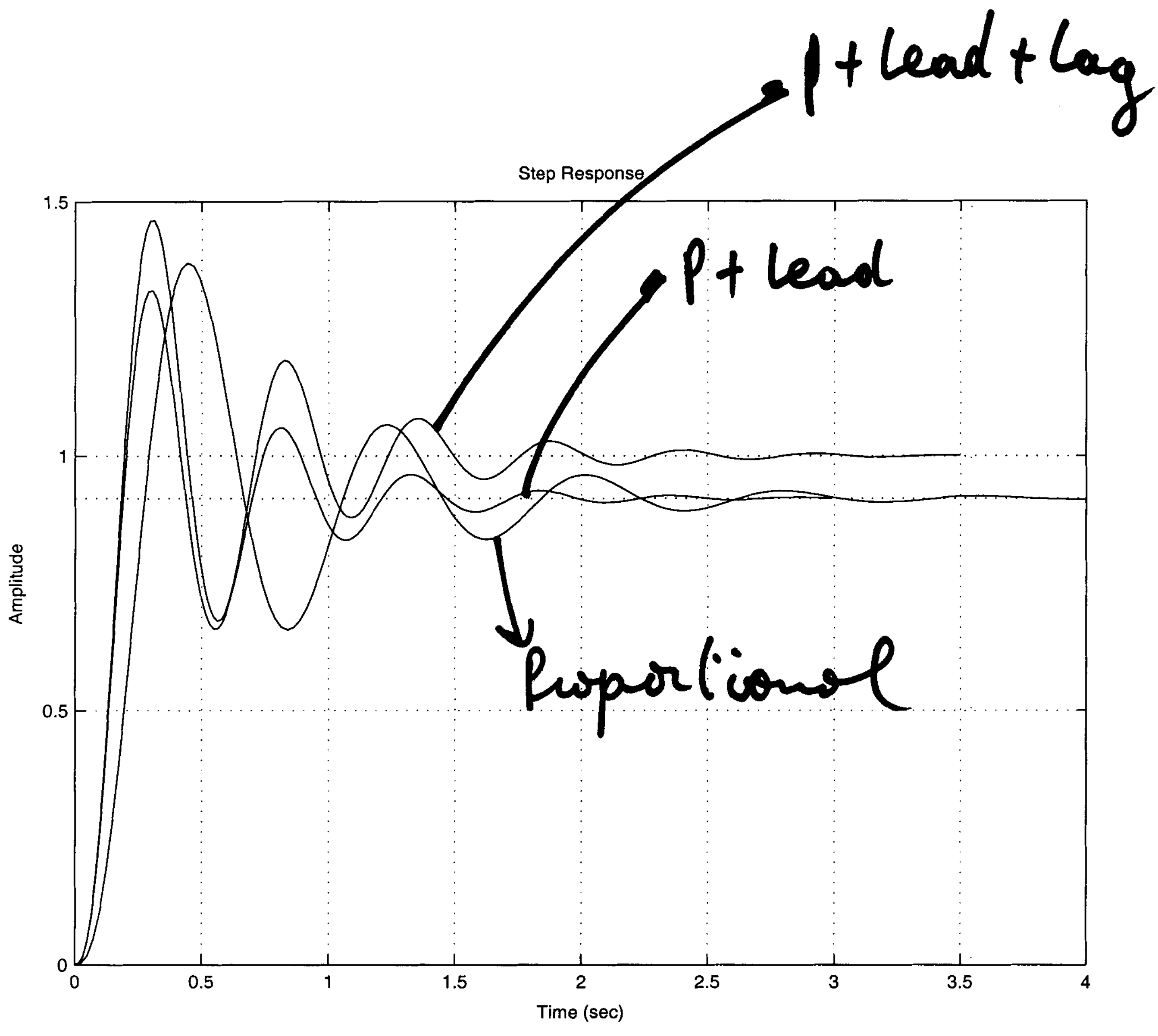


110x G(s)
110x Klead(s) x G(
P(jω)
Bode Diagram
Magnitude (dB)
40
20
0
-20
-40
-60
Phase (deg)
0
-45
-90
-135
-180
-225
-270
10^-1
10^0
10^1
10^2
Frequency (rad/sec)

$d = 12$

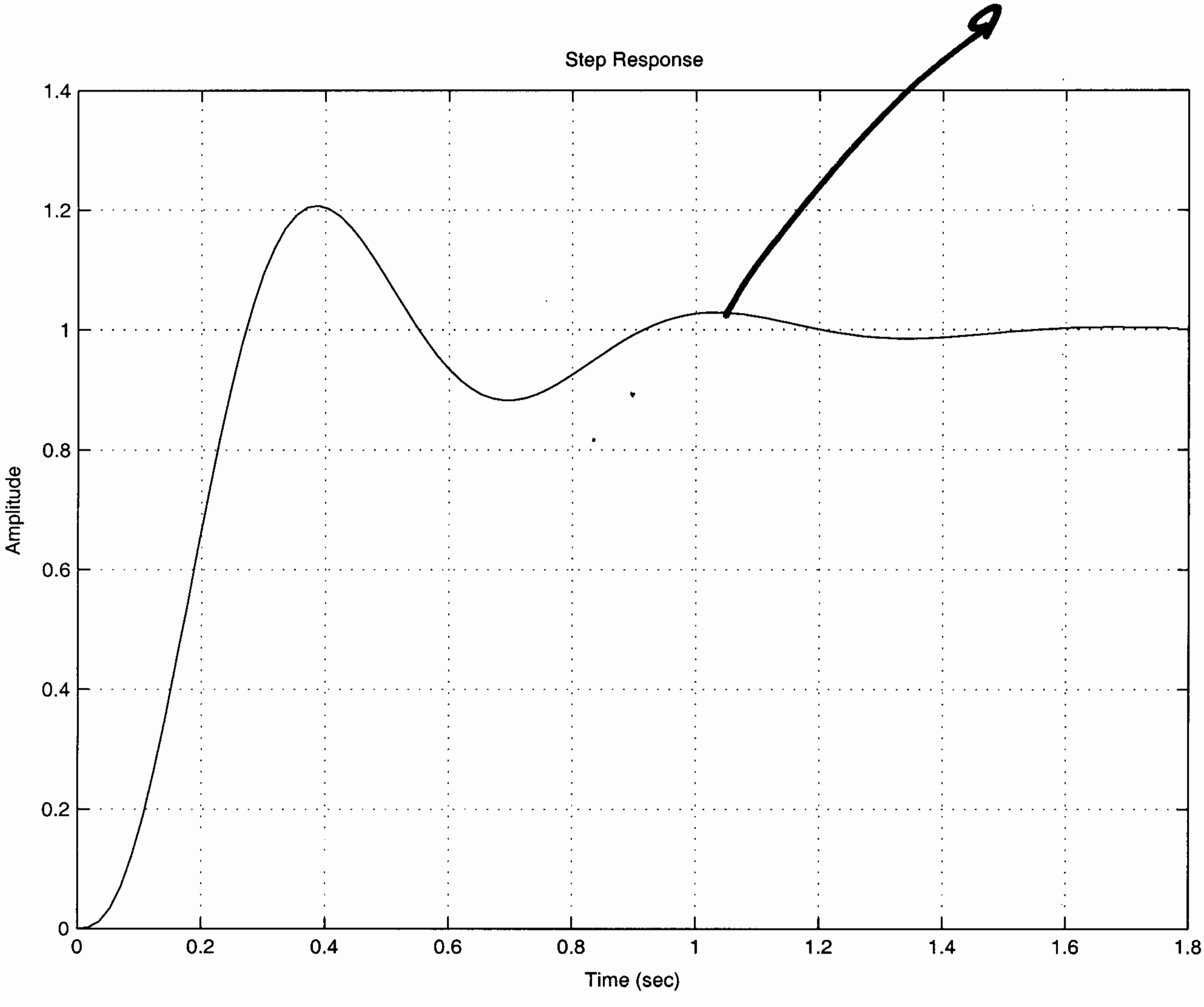
$c = 4.$

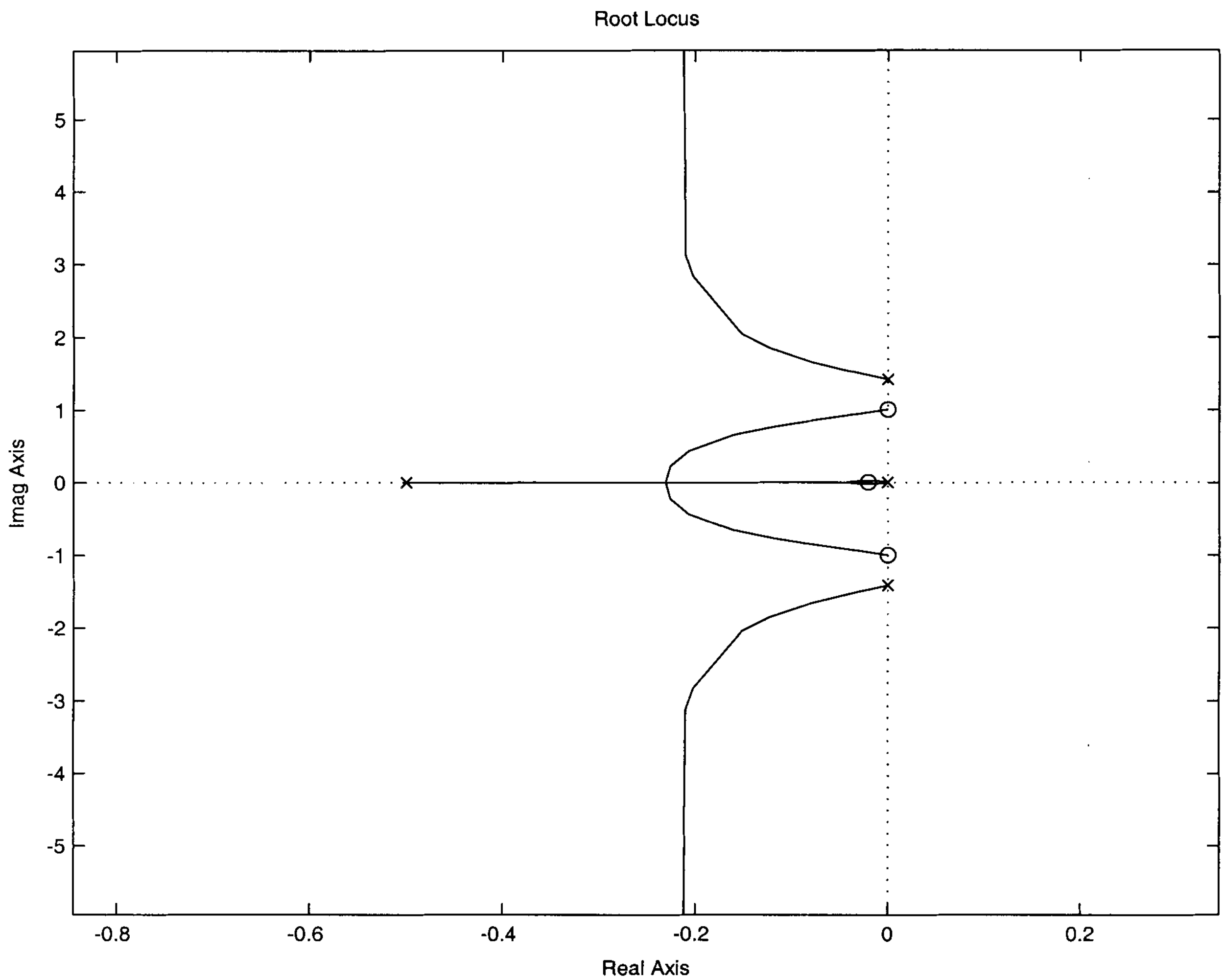

$10 \times K_{lead}(s) \times K_{lag}(s) \times G(s)$
$P(j\omega)$
$\ell^{-1}(j\omega)$
Bode Diagram
Magnitude (dB)
60
40
20
0
-20
-40
-60
Phase (deg)
0
-45
-90
-135
-180
-225
-270
phase!
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)



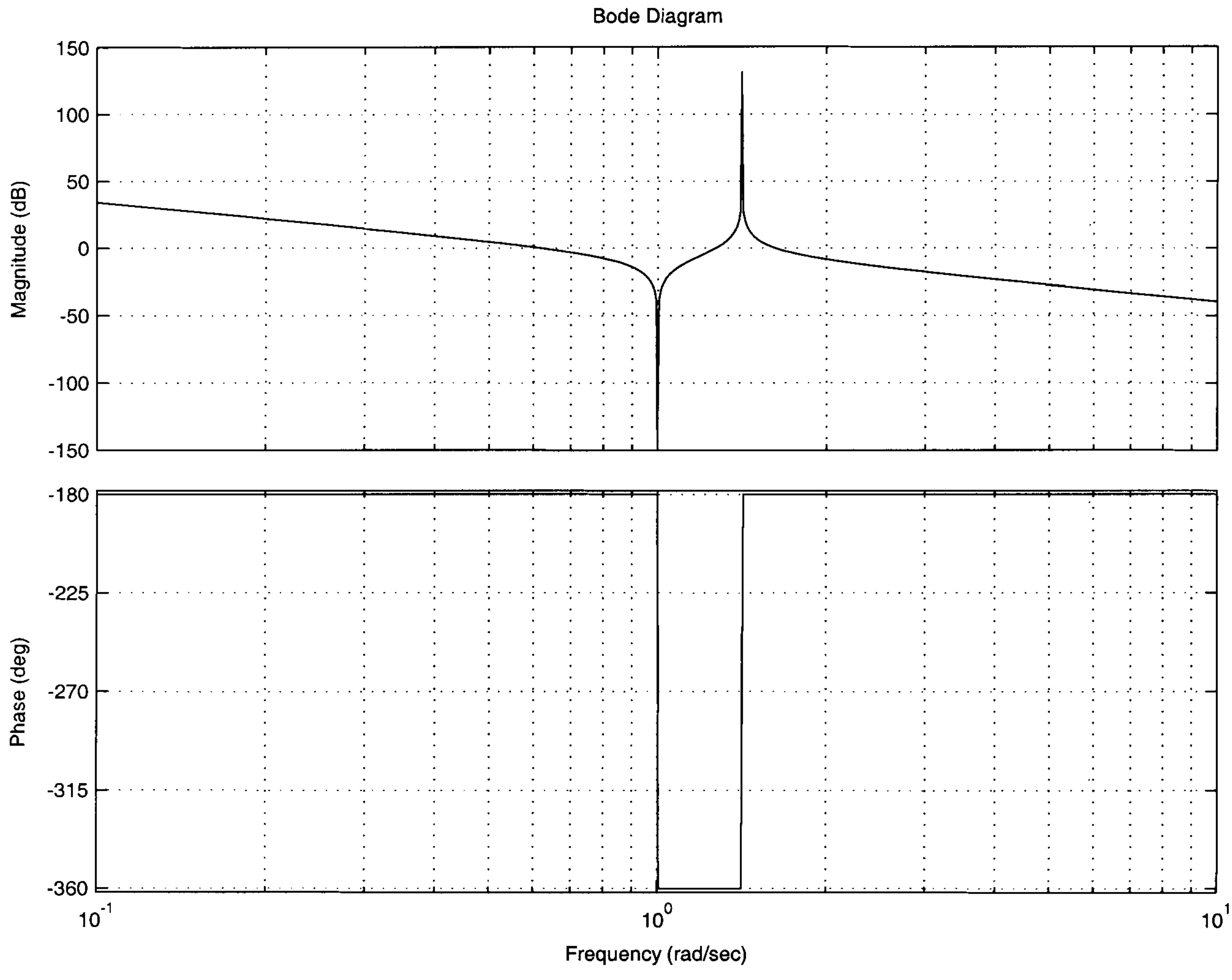
P + Lead + Lag
Step Response
P + Lead
1.5
1
0.5
0
Amplitude
Proportional
0
0.5
1
1.5
2
2.5
3
3.5
4
Time (sec)

K :110 → 66

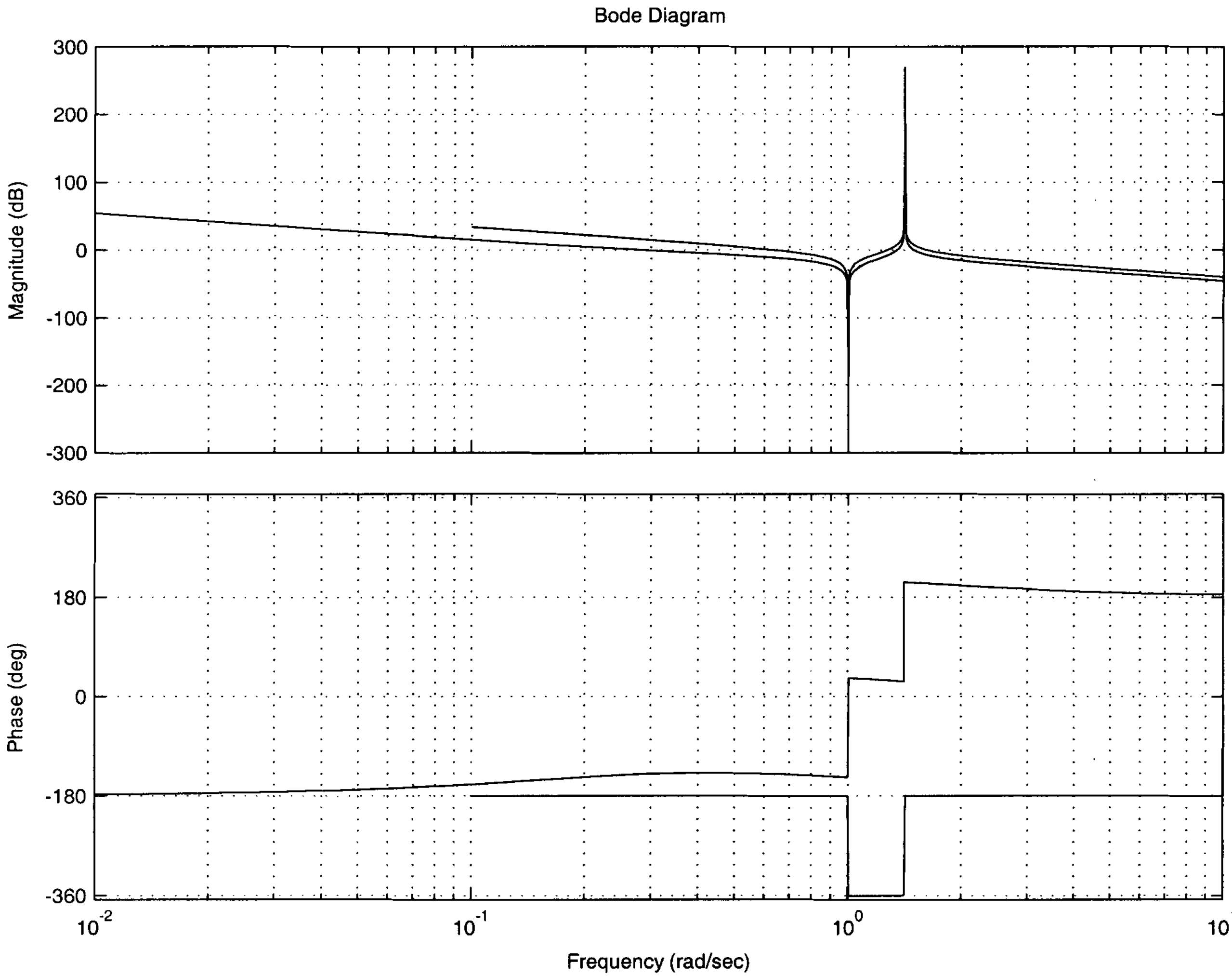




Root Locus
5
4
3
2
1
0
-1
-2
-3
-4
-5
Imag Axis
-0.8
-0.6
-0.4
-0.2
0
0.2
Real Axis



Bode Diagram
150
100
50
0
-50
-100
-150
Magnitude (dB)
-180
-225
-270
-315
-360
Phase (deg)
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)



$$\frac{s/0.02 + 1}{s/2 + 1}$$

$K = 0.2$