16.30/31 September 17, 2010
Prof. J. P. How and Prof. E. Frazzoli **Due:** September 24, 2010
T.A. B. Luders

16.30/31 Homework Assignment #2

Goals: Review frequency domain analysis, design, and stability criteria.

1. Analyze the stability of the unity gain negative feedback systems described by the following open-loop transfer functions, using the (i) root locus method, (ii) Nyquist plot, and (iii) asymptotic Bode plot:

   (a) $G(s) = \frac{60(s/5+1)}{s(s/0.4+1)(s+1)}$

   (b) $G(s) = \frac{30(s+1)}{(s+0.1)(s-2)(s+8)}$

   (c) $G(s) = \frac{0.1(s+0.1)}{(s+5)(s^2+4)}$

   All three plots should be drawn by hand for each transfer function (though you may check your answers in Matlab). You should note when a particular method cannot be used to assess stability.

2. Consider the unity gain negative feedback system with open-loop transfer function given by

$$G(s) = G_p(s)G_f(s), \quad G_p(s) = \frac{10s+1}{(s+1)(s/3.16+1)^2}, \quad G_f(s) = \frac{K}{s/p+1},$$

   where $G_p(s)$ is the plant and $G_f(s)$ is a low-pass filter applied to the plant with parameters $K, p \in \mathbb{R}$. In the questions that follow, $\bar{K} = 1$ and $\bar{p} = 10$.

   (a) Suppose $K = \bar{K}$ and $p = \bar{p}$; sketch by hand the asymptotic Bode plot. Use the approximation $\log_{10} 3.16 \approx 0.5$.

   Use your sketch from part (a) to answer parts (b)-(d) below.

   (b) Suppose $K = \bar{K}$ and $p = \bar{p}$; identify the phase margin and gain margin. What does this imply about the stability of the closed-loop system?

   (c) Suppose $p = \bar{p}$, but K is allowed to take any value such that $K > 0$. How does the Bode plot change as K is varied? Select K such that the phase margin is $20°$.

   (d) Suppose $K = \bar{K}$, but p is allowed to take any value such that $p > \bar{p}$. How does the Bode plot change as p is varied? Select p such that the phase margin is $20°$.

   (e) Repeat parts (b)-(d) in Matlab, using `sisotool`, and note any differences.

3.

Problem 8.10 removed due to copyright restrictions.
Van de Vegte, John. *Feedback Control Systems*.
3rd ed. Prentice Hall, 1993. ISBN: 9780130163790.

4. (16.31 required/16.30 extra credit). In some aerospace applications, the vehicle open-loop dynamics exhibit so-called "droop" in closed-loop performance when feedback is applied. Droop refers to poor closed-loop command following in the low- and mid-frequency ranges; high-frequency considerations can make this difficult to remedy.

   Consider the vehicle dynamics with transfer function

$$G(s) = \frac{s+10}{(s+100)(s^2+10s+1600)} G_{TD}(s),$$

   where $G_{TD}(s) = e^{-Ts}$ models a time delay of $T = 0.02$ seconds. Using Matlab, design a compensator that achieves less than 10% error in command following for signals with frequency content up to 0.5 rad/sec. If possible, try to extend this frequency range further - up to 5 rad/sec, or even larger. Motivate your compensator design, and plot the following:

   - [i] Open-loop Bode plot of your design, with all the compensator poles labeled;
   - [ii] Closed-loop Bode plot, indicating frequency range which meets the specification;
   - [iii] Closed-loop unit step response.

   Consider modeling the time delay as a first-order Padé approximation:

$$e^{-Ts} \approx \frac{1 - Ts/2}{1 + Ts/2}.$$

   See Section 5.7.3 in FPE for more information on incorporating time delays.