16.36 Communication Systems Engineering

Spring 2009

Name:

16.36 Communication Systems Engineering
Quiz -1
March 5, 2009

Part 1: Quick Questions (50 points; 5 points each)

Please provide brief explanations for your answers in order to receive full credit.

1. Time-shifting a signal changes its frequency composition.
   a. True
   b. False

2. A signal x(t) with bandwidth W, is sampled at a sampling rate 2W. Can the following be used to reconstruct the original signal from its samples?

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_S)k(\frac{t}{T_S} - n) \text{ where, } k(t) = \begin{cases} \text{Sinc}(t) & t \in [-1,1] \\ 0 & \text{otherwise} \end{cases}$$

   Explain your answer.

3. When conditioning on some event Y, the entropy of a random variable X will
   a. Increase
   b. Decrease
   c. Remain unchanged

   Please circle all possible answers and explain your choices briefly.

Name:

4. When designing a uniform quantizer for a Gaussian source, the mid-point of the quantization regions are the optimal quantization levels to minimize distortion.
    a. True
    b. False

5. The entropy of a uniform quantizer is always equal to the logarithm of the number of quantization levels.
    a. True
    b. False

6. Which of the following lengths of codewords are feasible for prefix-free codes:
    a. {2,2,3,3,3,3,4}
    b. {1,2,4,4,4,5}
    c. {2,2,2,3,4,4}

    Circle all that apply and briefly explain your choices.

Name:

7. For which of the following source alphabet probabilities can one construct a Huffman code with average codeword length that is equal to the source entropy
    a. {1/4, 1/4, 1/8, 1/8, 1/8, 1/8}
    b. {1/2, 1/8, 3/8}
    c. {1/4, 1/4, 1/4, 1/4}

    Circle all that apply and briefly explain your choices.

8. Given a band limited signal, x(t), with Bandwidth W, that is modulated by a carrier of frequency fc (i.e., $u(t) = x(t)\cos(2\pi fct)$). At what frequency must the signal, u(t), be sampled for perfect reconstruction?
    a. 2fc
    b. 2fc+2W
    c. 2fc+W
    d. 2W

9. A Gaussian source with zero mean and variance 10, is to be quantized using a uniform quantizer with 8 quantization levels. (See attached table).
    a. What is the resulting distortion?
    b. What is the spacing between quantization levels?
    c. What is the resulting entropy?

Name:

10. A source is encoded using the Lempel-Ziv algorithm with four bit code words. The encoded sequence is given by:

    0001 0000 0101 0110

    Using the above information reconstruct the dictionary and find the original bit sequence.

Question 2: Modulation (30 points)

Suppose you want to construct a 16-PAM modulator, using the basic pulse, g(t) given below.

$$g(t) = \begin{cases} A & t \in [0, T] \\ 0 & \text{otherwise} \end{cases}$$

Frequency planning regulations require that the two-sided null-to-null bandwidth of g(t) does not exceed 8 kHz.

a) What is the maximum information rate R_B that can be supported with 16-ary PAM (M=16)?

Name:

b) The baseband signal, g(t) is modulated by a sinusoidal carrier at frequency 2.4 MHz. Sketch an approximate frequency representation of the modulated signal. Clearly label magnitude and frequencies.

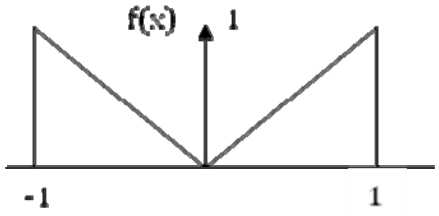
c) Consider next a symmetric 16-QAM modulator.
    i. Sketch the signal constellation plot, and label the points with their amplitude levels.
    ii. For each signal point give the associated signal energy level

Name:

Question 3: Quantization (20 points)

Suppose that the individual samples form independent, identically distributed random variables are distributed between -1 and 1 according to the pdf $f_x(x)$ given below:

$$f_x(x) = \begin{cases} |x| & -1 \le x \le 1 \\ 0 & otherwise \end{cases}$$



Suppose that you quantize this source using a three level quantizer (three quantization regions). Also assume that the quantization regions are chosen to be: [-1,-1/3], [-1/3,1/3], and [1/3, 1].

a) What would you choose as the quantized values for each of the regions in order to minimize average distortion?

b) What is the resulting binary entropy of the quantized source?