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# Lecture 12: Link Budget Analysis and Design 

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## Signal attenuation



- The signal suffers an attenuation loss $L$
- Received power $P_{R}=P_{T} / L$
- Received SNR = $\mathrm{E}_{\mathrm{b}} / \mathrm{N}_{0}, \mathrm{E}_{\mathrm{b}}=\mathrm{P}_{\mathrm{R}} / \mathbf{R}_{\mathrm{b}}$
- Antennas are used to compensate for attenuation loss
- Capture as much of the signal as possible

$\mathrm{L}=$ free space loss, $\mathrm{d}=$ distance between Tx and Rx
$\lambda=$ signal wavelength


## Antenna Gains

$$
\mathrm{G}_{\mathrm{R}}=\mathrm{A}_{\mathrm{R}} 4 \pi / \lambda^{2}
$$

$A_{R}$ is the effective area of the antenna
For Parabolic antenna $A_{R}=\pi \eta D^{2 / 4}$
$\eta=$ illumination efficiency factor, $0.5<\eta<0.6$
$\mathrm{D}=$ dish diameter
$\Rightarrow G_{R}=\eta(\pi D / \lambda)^{2}$
$\Rightarrow P R=P_{T} G_{T} D^{2} \eta /(4 d)^{2}$

## Antenna Beamwidth



- Beamwidth is a measure of the directivity of the antenna
- Smaller beamwidth concentrated power along a smaller area
- Free space loss assumes that power is radiated in all directions
- An antenna with a smaller beamwidth concentrates the power hence yields a gain
- For parabolic antenna, $\theta_{B} \sim 70 \lambda / D$
- Gain ( $G_{T}$ ) s proportional to $\left(\theta_{B}\right)^{-2}$
- Hence a doubling of the diameter $D$ increases gain by a factor of 4


## Example (GEO Satellite)

$$
\begin{aligned}
& \mathrm{d}=36,000 \mathrm{~km}=36,000,000 \text { meters } \\
& \mathrm{f}_{\mathrm{c}}=4 \mathrm{Ghz} \Rightarrow \lambda=0.075 \mathrm{~m} \\
& \mathrm{P}_{\mathrm{T}}=100 \mathrm{w}, \mathrm{G}_{\mathrm{T}}=18 \mathrm{~dB} \\
& \text { Receiver antenna is parabolic with } \mathrm{D}=3 \text { meters }
\end{aligned}
$$

A) What is PR?
B) Suppose $\left(E_{b} / N_{0}\right)_{\text {req }}=10 \mathrm{~dB}$, what is the achievable data rate $R_{b}$ ?

## Repeaters



- A repeater simply amplifies the signal to make up for attenuation
$P_{R 1}=P_{T} / L, P_{T 2}=P_{R 1} A, P_{R 2}=P_{T 2} / L, \ldots$
$P_{N 1}=P_{N}, P_{N 2}=P_{N 1} A / L+P_{N}, \ldots .$.
Let $A=L=P_{R K}=P_{T} / L, P_{N K}=K P_{N}$
$\mathrm{P}_{\mathrm{RK}} / \mathrm{P}_{\mathrm{NK}}=\mathrm{P}_{\mathrm{T}} / L K \mathrm{P}_{\mathrm{N}}=1 / \mathrm{K}\left(\mathrm{P}_{\mathrm{R} 1} / \mathrm{P}_{\mathrm{N} 1}\right)$
Received SNR is reduced by a factor of $K$

$$
\left(\mathrm{E}_{\mathrm{b}} / \mathrm{N}_{0}\right) \mathrm{k}=1 / \mathrm{K}\left(\mathrm{E}_{\mathrm{b}} / \mathrm{N}_{0}\right)
$$

## Regenerators



- A regenerator demodulates, detects and retransmits the signal
- Each segment has the same $P_{R} / P_{N}$ and the same received $E_{b} / N_{0}$
- $\quad P_{b}=$ probability of error on a segment (independent between segments)
- $P_{b}$ (overall) $=1-P($ no error $)=1-\left(1-P_{b}\right)^{K} \sim K P_{b}$
- Now compare repeater to regenerator (e.g. PAM)

$$
\begin{aligned}
& P_{b}=Q\left(\sqrt{2 E_{b} / N_{0}}\right) \\
& \text { For repeater : } \quad P_{b}(\text { overall })=Q\left(\sqrt{2 E_{b} / K N_{0}}\right) \\
& \text { For regenerator : } P_{b}(\text { overall })=K Q\left(\sqrt{2 E_{b} / N_{0}}\right)
\end{aligned}
$$

$K Q\left(\sqrt{2 E_{b} / N_{0}}\right)<Q\left(\sqrt{2 E_{b} / K N_{0}}\right)$

## Satellite example

- Uplink received $\left(E_{b} / N_{0}\right)_{u}=$ downlink received $\left(E_{b} / N_{0}\right)_{d}=10 d B$
- PAM modulation $P_{b}=Q\left(\sqrt{2 E_{b} / N_{0}}\right)$
- Repeater: Received $\left(E_{b} / N_{0}\right)_{u / d}=1 / 2\left(E_{b} / N_{0}\right)_{u}=10 \mathrm{~dB}-3 \mathrm{~dB}=7 \mathrm{~dB}$
$-\quad=>\mathrm{Pb}=5 \times 10^{-4}$ from table 7.55 or 7.58
- Regenerator: $P_{b}(u p)=P_{b}($ down $)=3 \times 10^{-6}$
- (from table with $\left.\left(E_{b} / N_{0}\right)_{d}=10 d B\right)$
- Hence $P_{b}$ (up/down) ~ $2 P_{b}($ up $) \sim 6 \times 3 \times 10^{-6}$
- Two orders of magnitude difference between repeaters and regeneration
- Greater difference with more segments

