### 16.50 Propulsion Systems HWK \#1

## Preliminary design of a Satellite launcher

We wish to produce a first-order design of a 3 -stage launch vehicle to place a 3 kg . nanosatellite in a circular equatorial of 500 km . altitude. Launch will be from the Equator, in an Easterly direction. All stages will use solid propellant rocket motors with $I_{s}=270 \mathrm{~s}$, and each will have a structural mass fraction $\varepsilon_{i}=M_{s i} / M_{0 i}=0.1$ (i=1,2,3).

For a first approximation to the required stage $\Delta V^{\prime}$ 's, assume two instantaneous impulses, one delivered near the ground ( $r=R_{E}=6730 \mathrm{~km}$ ) at an average elevation angle $\alpha=20^{\circ}$, that will place the vehicle in an ascent trajectory with apogee at 500 km altitude, and the other at the apogee, to add enough velocity to reach orbital conditions. Use conservation of angular momentum and energy (kinetic plus potential) in the ascent trajectory to find the initial velocity $v_{1}$ after the first impulse and the apogee velocity $v_{a}$. The velocity $v_{1}$ will be regarded as made up of the sum of the two velocity increments $\Delta V_{1}+\Delta V_{2}$ of the first and second stages, plus the Earth rotation velocity $\omega_{E} R_{E} \cos \alpha$, minus the full gravity loss $\Delta V_{G}$ and the full drag loss $\Delta V_{D}$. This determines $\Delta V_{1}+\Delta V_{2}$; assume $\Delta V_{1}=\Delta V_{2}$ and calculate both.

To estimate the gravity loss, assume a variation of the elevation angle $\gamma$ such that $\sin \gamma$ varies linearly in time during the first stage burn, from 1 (vertical launch) to 0 . Of course, the $20^{\circ}$ assumed for $\alpha$ above is a rough approximation of the average of $\gamma$ during both, the first and second stage burns. To calculate the stage firing times, assume each motor provides an initial trust acceleration $F_{i} / M_{0 i}=3 \mathrm{~g}$. One other piece of information for this time calculation is the propellant mass which itself depends on the stage $\Delta V$; this is not really known until the gravity loss is estimated, but you can iterate a bit, or make a simple first cut for this purpose only.

The calculation of the drag loss is more involved, and depends on a trajectory calculation, plus the aerodynamic characteristics of the vehicle. For now, assume $\Delta V_{D}=150 \mathrm{~m} / \mathrm{s}$.

With these assumptions, or some reasonable modification you may prefer (but if so, the new assumptions should be clearly stated and justified in some manner), calculate the initial mass of each of the stages, as well as their structural and propellant masses. Note that the payload for each stage is the initial mass of the next stage, except for the third stage, whose payload is the overall payload. Calculate also the firing time and the thrust of each motor.

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### 16.50 Introduction to Propulsion Systems

Spring 2012

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