

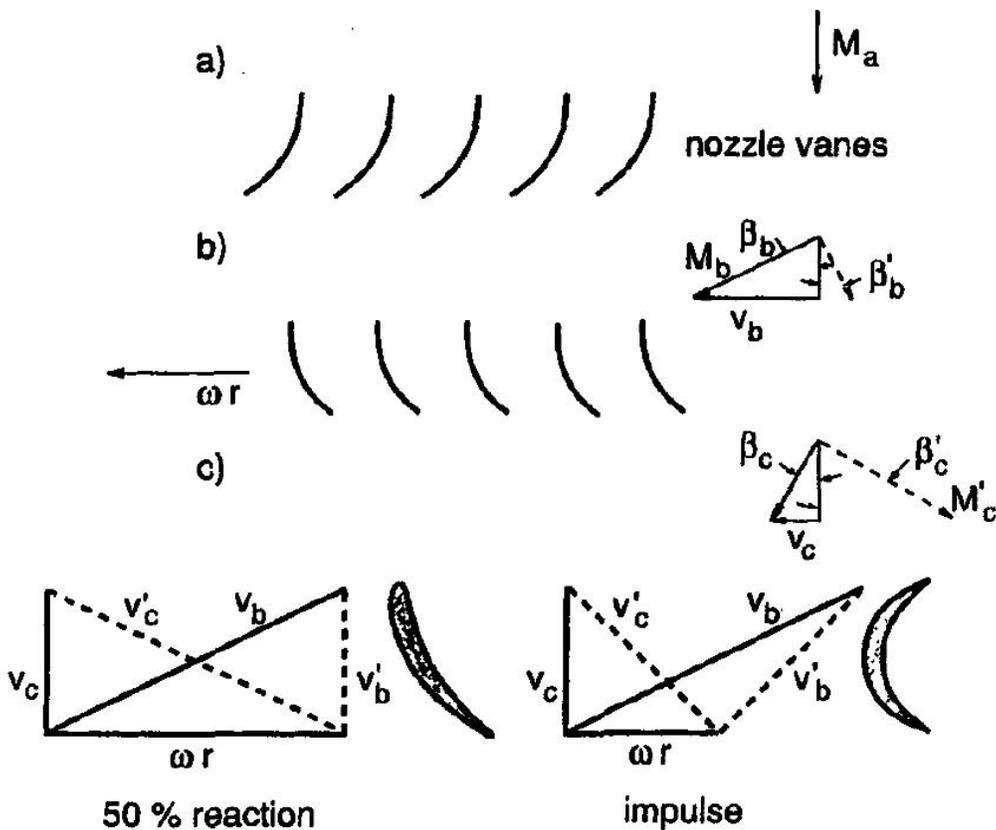
16.50 Lecture 27

Subjects: Turbines; Stage characteristics; Degree of reaction

Turbines behave according to the same principles of dynamic energy exchange as compressors. However they differ from compressors in some important ways. First since they extract energy from the flow rather than adding it, the pressure drops throughout a turbine. This leads to different fluid mechanical limitations than for compressors. Second, because of the thermodynamic requirements, they operate in gases at high temperatures, so that materials and cooling requirements are of central importance. We will begin by exploring the fluid-mechanical energy exchange in the turbine.

Euler Equation:

The exchange of energy is described by the Euler equation, just as for compressors. We will discuss here only axial flow turbines such as are generally used in aircraft engines, but it is important to realize that turbines in fact come in a wide variety of types, from extremely large hydraulic turbines for hydroelectric power generation to the familiar lawn sprinkler.



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Axial flow turbines use a stator blade row (usually called a nozzle row) followed by a rotating blade row that extracts the energy (often called the buckets). The nozzles

expand the flow to a high velocity and turn it, imparting an angular momentum to the flow.

The buckets generally further expand the flow, and turn it back toward the axial direction. The combination of the two blade rows is called a Turbine Stage.

Here we use the alphabetical notation to avoid confusion with the numerical station notation for the engine. Station a is equivalent to station 4 in the engine layout, and for a single stage turbine, station c should be equivalent to station 5 in the engine. A general stage layout is indicated at the top and two specific blade types at the bottom.

From the Euler equation, for the general situation,

$$c_p (T_{tc} - T_{tb}) = \omega(v_c r_c - v_b r_b)$$

$$1 - \frac{T_{tc}}{T_{tb}} = \frac{\omega r_c}{c_p T_{tb}} \left(\frac{r_b}{r_c} v_b - v_c \right)$$

Let us assume $r_b/r_c = 1$, and also that $v_c = 0$. The latter is desirable if the rotor is the last stage of a turbine, in order that there not be swirl in the flow entering the jet nozzle.

$$1 - \frac{T_{tc}}{T_{tb}} = 1 - \tau_t = \frac{(\omega r_b)}{c_p T_{tb}} v_b$$

Writing this in terms of Mach numbers and the flow angles indicated in the diagram, with $v_b = V_b \sin \beta_b = \sqrt{\gamma R T_b} M_b \sin \beta_b$, and $\omega r_b = \sqrt{\gamma R T_b} M_T$,

$$1 - \tau_t = \frac{(\gamma - 1) M_T M_b \sin \beta_b}{1 + \frac{\gamma - 1}{2} M_b^2} \quad (\text{No exit swirl})$$

Here M_T is the tip Mach number of the blades and M_b is the flow Mach number.

From this we can see that we want large M_T and large $M_b \sin \beta$ for large work per stage.

Degree of Reaction

As for the compressor there are many possible design choices for the turbine. One key choice is the relative amounts of pressure drop in the nozzles and buckets. This is best characterized in terms of the Degree of Reaction, which we will define as the ratio of static enthalpy change in the rotor to that in the rotor plus stator (stage):

$$R = \frac{h_b - h_c}{(h_b - h_c) + (h_a - h_b)}$$

We now use energy conservation in the moving frame of each element:

$$h_b + \frac{1}{2} V_b'^2 = h_c + \frac{1}{2} V_c'^2; \quad h_a + \frac{1}{2} V_a'^2 = h_b + \frac{1}{2} V_b'^2$$

to obtain the alternative interpretation

$$R = \frac{(V_c')^2 - (V_b')^2}{(V_c')^2 - (V_b')^2 + V_b'^2 - V_a^2} = \frac{\text{change in KE in rotor}}{\text{Total change in KE}}$$

Then, for constant axial velocity.

$$R = \frac{\sec^2 \beta_c' - \sec^2 \beta_b'}{(\sec^2 \beta_c' - \sec^2 \beta_b' + \sec^2 \beta_b' - 1)}$$

For zero exit swirl, i.e. $v_c = 0$.

$$v_c = \omega r - w \tan \beta_c' = 0; \quad \tan \beta_c' = \frac{\omega r}{w} = \frac{M_T}{M_b \cos \beta_b}$$

and with this relation we can eliminate β_c' from the expression for R.

Similarly,

$$w \tan \beta_b' = w \tan \beta_b - \omega r; \quad \tan \beta_b' = \tan \beta_b - \frac{\omega r}{w} = \tan \beta_b - \frac{M_T}{M_b \cos \beta_b}$$

Using $\sec^2 = 1 + \tan^2$ we can now eliminate β_b' and find

$$R = 1 - \frac{M_b \sin \beta_b}{2M_T}$$

and using this to eliminate $M_b \sin \beta_b$ in the expression for the temperature ratio

$$1 - \tau_t = \frac{(\gamma-1)M_T^2}{1 + \frac{\gamma-1}{2}M_b^2} \quad 2(1-R) \quad (\text{zero exit swirl})$$

From this result we see that for a value of M_T limited by stresses and the temperature and for a given value of M_b , the temperature drop in the turbine, hence its work per unit of mass flow, is related to R. As R increases the work decreases.

An alternative often used representation is in terms of the so-called flow and power coefficients, ϕ and ψ . The definitions are

$$\phi = \frac{w}{\omega r}; \quad \psi = \frac{\Delta h_t}{(\omega r)^2} = \frac{v_b - v_c}{\omega r}$$

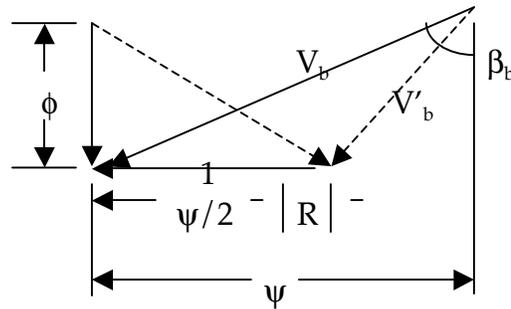
For zero exit swirl, $v_c=0$, we can also put $\psi = v_b / (\omega r) = M_b \sin \beta_b / M_T$, so that

$$R = 1 - \frac{\psi}{2} \quad \text{or} \quad \psi = 2(1 - R)$$

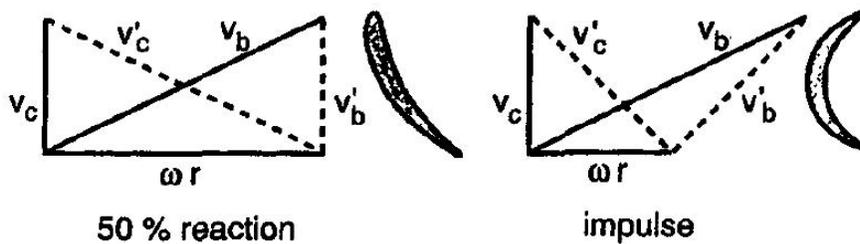
which very directly shows how the stage power decreases with the degree of reaction. In terms of flow coefficient and stator exit angle,

$$R = 1 - \frac{1}{2} \phi \tan \beta_b$$

These coefficients can all be readily visualized from a scaled velocity triangle in which the blade speed ωr is taken to be unity:



Drawing the velocity triangles for $R=1/2$ (called 50% reaction) and for $R=0$ (called Impulse),



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In the 50% reaction turbine the pressure drop in the moving blades equals that in the nozzles, while in the impulse turbine there is no pressure drop in the buckets. It all takes place in the nozzles.

In designing the turbine we have some latitude in choosing the degree of reaction R . The tangential velocity of the blades, hence M_T , is limited by the strength of materials at high temperature, so it may seem that we would like to use small R to maximize the temperature drop per stage. But the efficiency of the turbine decreases as R is decreased from $R=1/2$ toward $R=0$. The reason is that the boundary layers have more tendency to separate in the moving blades for $R=0$ than for $R=1/2$ because for $R=0$ there is no pressure drop as the flow is deflected.

Radial variations:

Just as for the compressor, the requirement for a constant temperature drop from hub to tip across the turbine flow path, places constraints on the degree of reaction. Since M_T is proportional to radius, it is larger at the tip than at the hub of the blades, so if M_b is about constant, R must decrease from the tip to the hub. Thus typically if the degree of reaction is near $1/2$ at the tip it may be considerably smaller at the hub.

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