16.810

Engineering Design and Rapid Prototyping

Lecture 9

IG.AID Structural Testing

Instructor(s)

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1G.A1D Outline

- Structural Testing
 - Why testing is important
 - Types of Sensors, Procedures
 - Mass, Static Displacement, Dynamics
- Test Protocol for 16.810 (Discussion)
 - Application of distributed load
 - Wing trailing edge displacement measurement
 - First natural frequency testing



Data Acquisition and Processing 1G.AlD for Structural Testing

(1) <u>Sensor Overview</u>:

Accelerometers, Laser sensors, Strain Gages,

Force Transducers and Load Cells, Gyroscopes

(2) <u>Sensor Characteristics & Dynamics</u>:

FRF of sensors, bandwidth, resolution, placement issues

(3) Data Acquistion Process:

Excitation Sources, Non-linearity, Anti-Alias Filtering, Signal Conditioning

(4) Data Post-Processing:

FFT, DFT, Computing PSD's and amplitude spectra,

statistical values of a signal such as RMS, covariance etc.

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(5) Introduction to System Identification

ETFE, DynaMod Measurement Models

1G.R10 Why is Structural Testing Important?

- Product Qualification Testing
- Performance Assessment
- System Identification
- Design Verification
- Damage Assessment
- Aerodynamic Flutter Testing
- Operational Monitoring
- Material Fatigue Testing



DSP = digital signal processing

Example: Ground Vibration Testing



F-22 Raptor #01 during ground vibration tests at Edwards Air Force Base, Calif., in April 1999

1C.R10 I. Sensor Overview

This Sensor morphology is useful for classification of typical sensors used in structural dynamics.

Sensor Morphology Table

Туре	Linear		Rotationa		
Bandwidth	Low		Medium		High
Derivative	Position		Rate		Acceleration
Reference	Absolute		Relative		
Quantity	Force/Torque		Displacement		
Impedance	Low		High		

Example: uniaxial strain gage

Need units of measurement: [m], [Nm],[µstrain],[rad] etc...

1G.A1D Sensor Examples for Structural Dynamics



1G.AID Strain Gages



Strain gages measure strain (differential displacement) over a finite area via a change in electrical resistance $R=l\rho$ [Ω]

Current Nominal length I_o : $I_o = \frac{V_{in}}{l_o \rho}$

With applied strain: $I_{\varepsilon} = \frac{V_{in}}{(l_{o} + \Delta l)\rho}$

Implementation: Wheatstone bridge circuit

strain gages feature polyimide-encapsulated constantan grids with copper-coated solder tabs.

1G.RID Accelerometers

- Accelerometers measure linear acceleration in one, two or three axes. We distinguish:
 - single vs. multi axis accelerometers
 - DC versus non-DC accelerometers

Recorded voltage

$$V_{out}(t) = K_a \ddot{x}(t) + V_0$$



Can measure: linear, centrifugal and gravitational acceleration Use caution when double-integrating acceleration to get position (drift)

> Single-Axis Accelerometer must be aligned with sensing axis.

Example: Kistler Piezobeam (not responsive at DC)

Manufacturers: Kistler, Vibrometer, Summit,...

C1 C2

Example: Summit capacitive accelerometer (DC capable)



IC.RID Laser Displacement Sensors



(position sensitive device - PSD).

<u>Vibrometers</u> include advanced processing and scanning capabilities.

Manufacturers: Keyence, MTI Instruments,...

<u>Advantages:</u>

contact-free measurement

Disadvantages:

need reflective, flat target limited resolution ~ 1μm

1C.RID Force Transducers / Load Cells

Force Transducers/Load Cells are capable of measuring Up to 6 DOF of force on three orthogonal axes, and the moment (torque) about each axis, to completely define the loading at the sensor's location

The high stiffness also results in a high resonant frequency, allowing accurate sensor response to rapid force changes.

Load cells are electro-mechanical transducers that translate force or weight into voltage. They usually contain strain gages internally.

Manufacturers: JR3, Transducer Techniques Inc. ...



1G.RID Other Sensors

Fiber Optic strain sensors (Bragg Gratings)



- Ring Laser Gyroscopes (Sagnac Effect)
- PVDF or PZT sensors



1G.RIN II. Sensor Characteristics & Dynamics

Goal: Explain performance characteristics (attributes of real sensors)

When choosing a sensor for a particular application we must specify the following requirements:

Sensor Performance Requirements:

- Dynamic Range and Span
- Accuracy and Resolution
- Absolute or Relative measurement
- Sensor Time Constant
- Bandwidth
- Linearity
- Impedance
- Reliability (MTBF)

Constraints:

Power: 28VDC, 400 Hz AC, 60 Hz AC Cost, Weight, Volume, EMI, Heat



Calibration is the process of obtaining the S(X) relationship for an actual sensor. In the physical world S depends on things other than X. Consider modifying input Y (e.g. Temp)

<u>E.g. Load cell calibration data:</u> X= mass (0.1, 0.5 1.0 kg...) S= voltage (111.3, 563.2, 1043.2 mV)



Sensor Time Constant **1G.**A10

How quickly does the sensor respond to input changes ?

First-Order Instruments

$$a_1 \frac{dy}{dt} + a_o y = b_o u$$

Dividing by a_o gives:

$$\frac{a_1}{\underbrace{a_0}}\frac{dy}{dt} + y = \frac{b_o}{\underbrace{a_o}}_K u$$

In s-domain:

 $\frac{Y(s)}{K} = \frac{K}{K}$ $U(s) \quad \tau s + 1$

K: static sensitivity

Second-Order Instruments
$$d^2$$
 where d^2

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_o y = b_o u$$

Essential parameters are:

1G.RID Sensor Range & Resolution



Resolution = smallest input increment that givesTrise to a measurable output change. Resolutionmand accuracy are NOT the same thing !

Threshold= smallest measurable input

IGAID Accuracy



<u>Note:</u> Instrument standard used for calibration should be ~ 10 times more accurate than the sensor itself (National Standards Practice)

1G.alo Linearity



1G.AIn Placement Issues

Need to consider the <u>dynamics</u> <u>of the structure</u> to be tested before choosing where to place sensors:



Observability determined by product of mode shape matrix Φ and output influence coefficients β_v

Observability gramian:

 $W_o \to A^T W_o + W_o A + C^T C = 0$

Other considerations:

- Pole-zero pattern if sensor used for control (collocated sensor-actuator pair)
- Placement constraints (volume, wiring, surface properties etc...)

16.910 Invasiability / Impedance

How does the measurement/sensor influence the physics of the system?

Remember Heisenberg's uncertainty principle: $\Delta x \Delta p \ge \hbar$ Х $q_{i1} \cdot q_{i2} = P$ [W] sensor Power drain: $P = q_{i1}^2 / Z_{gi}$

Impedance characterizes "loading" effect of sensor on the system. Sensor extracts power/energy -> Consider "impedance" and "admittance"



<u>Load Cell:</u> High Impedance = k_s large vs. <u>Strain Gage</u>: Low Impedance = k_s small

Conclusion: Impedance of sensors lead to errors that must be modeled in a high accuracy measurement chain (I.e. include sensor impedance/dynamics) **Massachusetts Institute of Technology**

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1C.RID System Excitation Types

<u>Type A</u> : Impulsive Excitation (Impulse Hammers)

Type B: Broadband Noise (Electromechanical Shakers)

Type C: Periodic Signals (Narrowband Excitation)

Type D: Environmental (Slewing, Wind Gusts, Road, Test track, Waves)



1G.A1D Excitation Sources

 $u(t) = F(t) = F_o \delta(t)$ Wide band excitation at various energy levels can be applied to a structure using impulse force hammers. They generate a nearly perfect impulse.

$$u(t) \longrightarrow G(\omega) \longrightarrow y(t) \qquad y(t)$$

Impulse Response h(t)

Broadband

$$y(t) = \int_{-\infty}^{t} u(\tau)h(t-\tau)d\tau \quad \text{(convolution integral)}$$
$$Y(\omega) \stackrel{-\infty}{=} \underbrace{U(\omega)}_{F_o \cdot 1} H(\omega) \longrightarrow \begin{array}{c} G(\omega) = H(\omega) \\ \text{(no noise)} \end{array}$$

The noise-free response to an ideal impulse contains all the information about the LTI system dynamics

Shaker can be driven by periodic or broadband random current from a signal generator.

$$S_{yy}(\omega) = G(j\omega)S_{uu}(\omega)G^{H}(j\omega) \longrightarrow$$

Output PSD



Excitation amplitude selection is a tradeoff between introducing non-linearity (upper bound) and achieving good signal-to-noise ratio (SNR) (lower bound).

1C.AID Signal Conditioning and Noise When we amplify the signal, n(t) we introduce measurement noise n(t), which corrupts the Test y(t) **u(t)** Article

measurement y(t) by some amount.

<u>Power Content in Signal</u> Power Content in Noise = $\int_{-\infty}^{+\infty} \frac{S_{yy}(\omega)}{S(\omega)} d\omega$ Consider Signal to Noise Ratio (SNR) =

$$Y(s) = KG(s)U(s) + N(s)$$

Look at PSD's:

$$\frac{S_{yy}(\omega)}{S_{yy}(\omega)} = \left| KG(j\omega) \right|^2 + \frac{S_{nn}}{S_{uu}}$$

Solve for system dynamics via Cross-correlation uy

$$G(j\omega) \cong \frac{S_{uy}(\omega)}{S_{uu}(\omega)}$$

Quality estimate via $\rightarrow C_{yu}(\omega)$ coherence function

Decrease noise effect by: •Increasing $S_{\mu\mu}$ (limit non-linearity) •Increasing K (also increases S_{nn}) •Decreasing Snn (best option)

Noise contribution





<u>Nyquist Theorem:</u> In order to recover a signal x(t) exactly it is necessary to sample the signal at a rate greater than <u>twice</u> the highest frequency present.

Rule of thumb: Sample 10 times faster than highest frequency of interest !



1G.RID IV. Data Post-Processing

Goal: Explain what we do after data is obtained

Stationary processes (E[x], E[x ²],) are time invariant	Analyze in frequency-domain		
Transient processes	Analyze in time-domain (T _s , Percent overshoot etc.)		
Impulse response	Fourier transform of $h(t)$ -> $H(\omega)$		

The FFT (Fast Fourier Transform) is the workhorse of DSP (Digital Signal Processing).

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G.R10 Metrics for steady state processes



1G.A10Metrics for transient processes



1GLAID Metrics for impulse response/decay from Initial Conditions



IGAID FFT and DFT



Approximates the
continuous Fourier transform:
$$X(\omega) = \int_{0}^{T} x(t)e^{-i\omega t} dt$$
$$= 0,1,...,N-1$$
$$= 0,1,...,N-1$$
and r are integers
I = # of data points Note:
= time length of data X_k are complex

 $x(t) = \sum c_n e^{i\omega_n t}$

The Power Spectral Density (PSD) Function gives the frequency content of the power in the signal:

$$W_{k} = \frac{2\Delta T}{N} X_{k} \cdot X_{k}^{H}$$

FFT - faster algorithm if N = power of 2 (512, 1024,2048,4096)

Т

1G.**A1D** Amplitude Spectra and PSD

Example: processing of Laser displacement sensor data from SSL testbed



MATLAB coding 1G.A1nTime domain: Given signal x and time vector t, N samples, dt=const. dt=t(2)-t(1); % sampling time interval dt fmax=(1/(2*dt)); % upper frequency bound [Hz] Nyquist % time sample size [sec] T=max(t); N=length(t); % length of time vector (assume zero mean) x_mean=mean(x); % mean of signal x rms=std(x); % standard deviation of signal should match Amplitude Spectrum: X k = abs(fft(x)); % computes periodogram of x AS fft = (2/N)*X k; % compute amplitude spectrum k=[0:N-1]; % indices for FT frequency points f fft=k*(1/(N*dt)); % correct scaling for frequency vector f fft=f fft(1:round(N/2)); % only left half of fft is retained AS fft=AS fft(1:round(N/2)); % only left half of AS is retained Power Spectral Density: PSD fft=(2*dt/N)*X k.^2; % computes one-sided PSD in Hertz PSD fft=PSD fft(1:length(f fft)); % set to length of freq vector

1G.AID V. System Identification

Goal: Explain example of data usage after processing

Goal: Create a mathematical model of the system based on input and output measurements alone.



LGLAIDEmpirical Transfer FunctionEstimate (ETFE)

$$\hat{G}_{kl}(j\omega) = \frac{Y_k(j\omega)}{U_l(j\omega)}$$

$$\hat{G}_{kl}(j\omega) = G_{kl}(j\omega) + \frac{N(j\omega)}{\bigcup_{l} (j\omega)}$$

Estimated TF True TF True TF Noise

Obtain an estimate of the transfer function from the I-th input to the j-th output

What are the consequences of neglecting the contributions by the noise term ?

Compute:
$$S_{yu} = E[Y(s)U^*(s)]$$
 $S_{UU} = E[UU^*]$, $S_{YY} = E[YY^*]$

Quality Assessment of transfer function estimate via the coherence function:

$$C_{yu}^{2} = \frac{\left|S_{yu}\right|^{2}}{S_{yy}S_{uu}} \qquad C_{yu} \to 1$$
$$C_{yu} \to 0$$

Implies small noise (Snn ~ 0)

Implies large noise Poor Estimate

Typically we want C_{yu} > 0.8



IGLAID Model Synthesis Methods

ample: Linear Least Squares
$$G(s) = \frac{b_{n-1}s^{n-1} + \dots + b_o}{s^n + a_{n-1}s^{n-1} + \dots + a_o} = \frac{B(j\omega, \theta)}{A(j\omega, \theta)}$$

We want to obtain an estimate of the polynomial coefficient of G(s) $\theta^T = \begin{bmatrix} a_o & a_1 & \dots & a_{n-1} & b_o & \dots & b_{n-1} \end{bmatrix}$ Define a cost function: $J = \frac{1}{N} \sum_{k=1}^{N} \frac{1}{2} \begin{bmatrix} \hat{G}(j\omega_k) - \frac{B(j\omega_k, \theta)}{A(j\omega_k, \theta)} \end{bmatrix}^2$

J is quadratic in θ : can apply a gradient search technique to minimize cost J

Search for:
$$\frac{\partial J}{\partial \theta} = 0 \rightarrow \theta_{optim}$$

Simple method but two major problems
Sensitive to order n
Matches poles well but not zeros

Other Methods: ARX, logarithmic NLLS, FORSE

1G.RID State Space Measurement Models



Measurement models obtained for MIT ORIGINS testbed (30 state model)



Software used: DynaMod by Midé Technology Corp.

1G.Alo Summary

<u>Upfront work</u> before actual testing / data acquisition is considerable:

- What am I trying to measure and why ?
- Sensor selection and placement decisions need to be made
- Which bandwidth am I interested in ?
- How do I excite the system (caution for non-linearity) ?

The topic of signal conditioning is crucial and affects results :

- Do I need to amplify the native sensor signal ?
- What are the estimates for noise levels ?
- What is my sampling rate ΔT and sample length T (Nyquist, Leakage) ?
- Need to consider Leakage, Aliasing and Averaging

Data processing techniques are powerful and diverse:

- FFT and DFT most important (try to have 2^N points for speed)
- Noise considerations (how good is my measurement ? -> coherence)