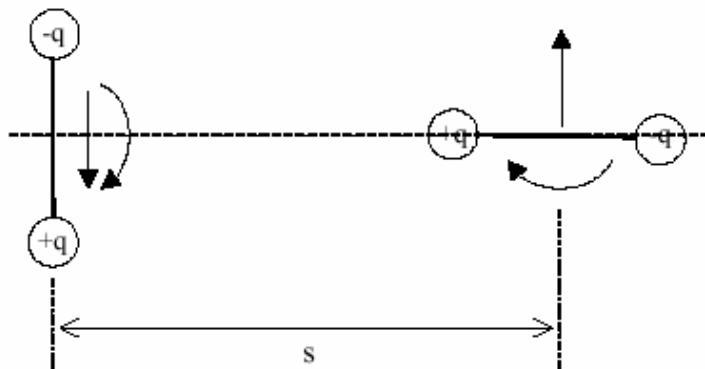## *E   Reaction Wheel Design (LW, WF)*
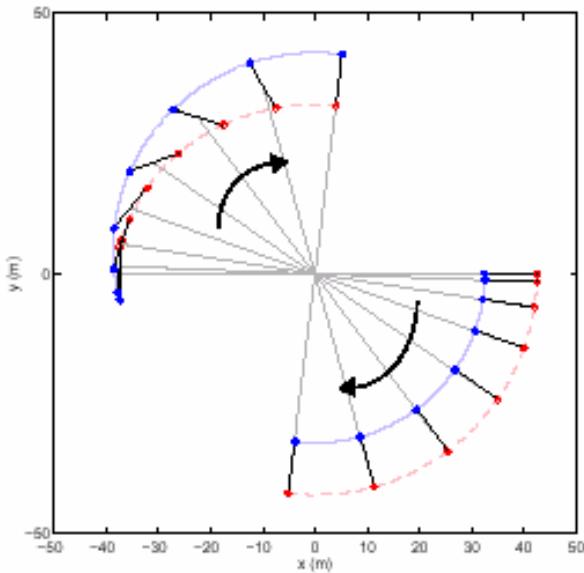
E.1   Role of RWA (LW)

The spin-up process requires equating the centripetal force on a vehicle with the electromagnetic force generated by the magnets. This process is depicted in Figure E.1-A and in Figure E.1-B. Figure E.1-A shows the initial configuration of the dipoles. Although the finalized design of the system uses two large, coreless electromagnets, the electromagnets are represented here as effective dipoles. The dipoles begin perpendicular to each other.

Figure E.1-A: Initial Dipole Configuration

Figure E.1-B depicts the two dipoles as they begin to spin up. As soon as a current is applied to the electromagnets, forces and moments are induced in the dipoles. The moments are due to the perpendicular geometry of the system at this point. These moments must be counteracted by applied torque from the RWA. The dipoles initially move in the directions of their respective net forces. As they begin to move, the applied torque from the RWA is decreased to induce centripetal motion of the system. As centripetal force increases, the applied torque continuously decreases, allowing the rotating dipoles to slowly align along the same axis. At the point where the dipoles are perfectly aligned and the centripetal force is equal to the total magnetic force, no more applied torque is necessary. At this point, the system is in steady-state rotation Figure E.1 -B shows the process of spin-up.

Figure E.1-B: Spin-up Configuration

The RWA consists of a flywheel and a motor. The motor powers the flywheel and must provide the necessary torque to balance the moments that are produced by the electromagnets. The spinning wheel stores the system's angular momentum, balancing the system and guiding the system through spin-up to steady-state rotation. The RWA functions on the principle of the conservation of angular momentum. Angular momentum is conserved because the spin-up maneuver is performed entirely using torques and forces that are internal to the system. Therefore, the sum of the angular momentum that accumulates in the wheels on the (in this case) two vehicles is equal and opposite to the angular momentum associated with the spinning of the system about its own center of mass.

E.2 Requirements

- Wheel
    - o As a minimum, the flywheel must store the angular momentum necessary to operate a two-vehicle system in a steady state maneuver. The vehicles will perform the maneuver at a separation distance (between vehicle centers) of two meters and a rotation rate of one rotation per minute.
    - o The flywheel may not be made from an electrically conducting material. The electromagnetic field in which the wheel must operate would induce eddy currents in any wheel made from conducting material. Eddy currents would act as a retarding force against the motion of the wheel and this would hurt the performance of the reaction wheel assembly.
    - o The maximum wheel diameter is limited by the total vehicle size to less than half of a meter.
    - o Due to the material properties of the flywheel the wheel must be capable of storing the necessary angular momentum while operating at a speed below seven thousand rotations per minute.

- Motor
    - o The motor in the RWA must provide sufficient torque to accelerate the flywheel in either direction in order to counter moments induced by the electromagnets.

E.3 Wheel Design (WF)

E.3.1 Material Selection

The design of the reaction wheel assembly was complicated with the decision from the Electro-magnet team to use hoops of super conducting wire to generate the necessary magnet forces to operate the vehicles. The reaction wheel assembly will be placed inside these hoops and therefore will be operating inside of a magnetic field. Due to problems with eddy currents, which act as strong resisting force to spinning metal wheels inside of a magnetic field, the flywheel cannot be made from any metallic materials. This forced the search for non-metallic materials with a sufficient density and structural rigidity to provide for the angular momentum storage of the vehicles. High-density urethanes appear to be the best solution. These materials provide the density and structural properties necessary to perform as needed and can also be molded into any shape for a relatively low cost. The reaction wheel assembly team has decided to use urethane wheels to be manufactured by Advanced Urethane Solutions because of their ability to meet all of our specifications at a low cost and with a reasonable delivery time.

E.3.2 Basic Geometry

The flywheel consists of a thick outer ring and thin inner disk. Moment of inertia is dependent on the total mass of material used and the distance of that material from the axis of rotation. The most efficient flywheel design places a majority of the flywheel mass far from the axis of Rotation - the center of the flywheel.
 Figure E.3-A depicts the basic geometry used as the starting point in the flywheel design.
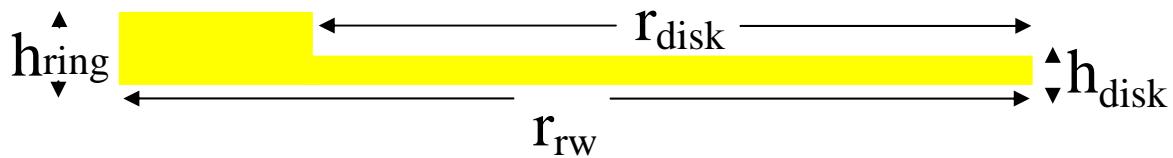

Figure E.3-A: Basic Geometry of Initial Flywheel Design

E.3.3 Governing Model

The flywheel on each vehicle must be capable of storing one half of the total angular momentum of the entire system. This is necessary to allow the system to spin up from a static equilibrium into a steady-state rotation, which is a maneuver that will be conducted for this project. The total angular momentum that must be stored in the reaction wheels must be equal to the angular momentum generated during the spin up of the vehicles.

$$H_{rw} = \frac{H_{tot}}{2}$$

Equation E.3-1

It is necessary to determine both the angular momentum that will be created by the system and the total angular momentum that can be stored by the reaction wheel. Angular momentum is the product of the mass moment of inertia and the angular rotation rate.

$$H = I\Omega$$

Equation E.3-2

The mass moment of inertia of the entire system is found by summing the contributions of each of the individual vehicles. The total mass moment of inertia of the system is found using the parallel axis theorem.

$$I_{tot} = 2(I_{veh} + m_{veh}(\frac{s}{2})^2)$$

Equation E.3-3

Where I_{veh} is the mass moment of inertia of an individual vehicle, m_{veh} is the mass of a vehicle, and s is the separation distance between two vehicles. It is not possible at this time to get an exact value for the mass moment of inertia for each vehicle (I_{veh}). However, it is possible to make estimates from the contributions of each of the vehicle components and sum those contributions. The mass moment of inertia for each of the major vehicle components can be found in Table E.3-A below.

Table E.3-A: Mass Moment of Inertia Values

Component	Mass Moment of Inertia
Outer electromagnet ring and container	.34 kg m^2
Inner electromagnet ring and container	.24 kg m^2
Batteries	.032 kg m^2
Metrology	.1 kg m^2
Air Pucks	.1 kg m^2
Other	.05 kg m^2

Using the results from Table E.3-A and adding a margin of error of 4.5%, the mass moment of inertia for each vehicle, about its center of mass, is approximately 0.9 kg m^2. The current test plan calls for the two vehicles to separated by a distance of 2 m and rotate at one RPM with a vehicle mass of 18 kg. Using this information in Equation E.3-2 gives the total angular momentum of the vehicle as 3.96 kg m^2/s.

The maximum velocity of the wheel is limited by the structural integrity of the material when spun at high angular velocities. For the current wheel design and material properties, the maximum speed of the wheel before structural failure is 15,000 RPM (see section 3.4 below) and before structural deformation is 7,000 RPM. In order to ensure the safety of the wheel and those operating the vehicle, we have set the design operating velocity for the flywheel to be 2000 RPM. For an operating velocity of 2000 RPM and the total angular momentum of 3.96 kg m^2/s calculated above, Equations E.3-1 and E.3-2 yield a necessary mass moment of inertia for the flywheel of 0.01 kg m^2.

The mass moment of inertia of the flywheel is dependent on the specific geometry of the wheel and can be found by treating the disk and ring separately and superimposing the results. From Figure E.1-B, the mass moment inertia of the disk is given by:

$$I_{disk} = \rho \frac{\pi}{2} h_{disk} r_{disk}^4$$

Equation E.3-4

The mass moment of inertia of the ring is given by:

$$I_{ring} = \rho \frac{\pi}{2} h_{ring} (r_{rw}^4 - r_{disk}^4)$$

Equation E.3-5

Summing Equations E.3-4 and E.3-5 yields the total mass moment of inertia for the flywheel.

$$I_{rw} = \rho \frac{\pi}{2} [h_{ring} (r_{rw}^4 - r_{disk}^4) + h_{disk} r_{disk}^4]$$

Equation E.3-6

In these equations, the geometric variables are as defined in Figure E.3-B and the density, ρ, is 1140 kg/m^3. The final geometry of the flywheel was determined using Equation E.3-6 and taking into consideration the manufacturability of the design. The final wheel design is specified in Figure E.3-B. This final geometry meets the design requirements and provides a wheel with a mass of 1.1 kg.
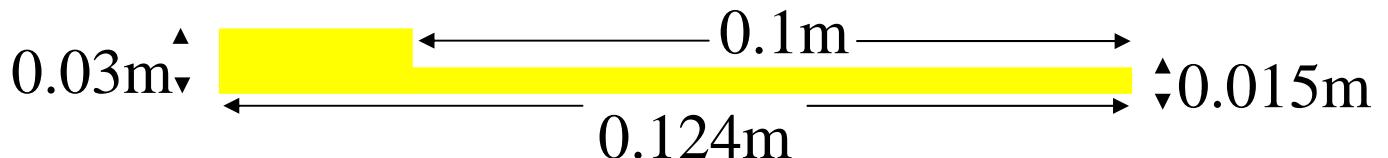

Figure E.3-B: Final Wheel Design

E.3.4 Structural Integrity

Disks spinning at high angular velocities can experience incredibly large stresses. In order to ensure the safety of our system, the flywheel must be capable of withstanding the stresses that it will experience during operation. The stresses for the flywheel are greatest in the lip of the ring. The maximum stress for a spinning ring is given by

$$\sigma_{max} = \frac{3+v}{4} \rho \omega^2 (R_O^2 + \frac{1-v}{3+v} R_I^2)$$

Equation 3.7-7

where ω is the spin rate of the system, $\mathbf{R_O}$ is 0.124m and $\mathbf{R_I}$ is 0.1m and v is the Poisson ratio of the material. For the urethane wheels we are using, the ultimate stress is 36 MPa and the yield stress is 9 MPa. Thus, the angular velocity at which the wheel will fail is 15,000 RPM and the rate at which the material will begin to yield is 7,000 RPM. In order to prevent failure, the system is designed to operate at 2,000 RPM and voltage limits will be placed on the power supplying the motor to prevent it from spinning faster than 5,000 RPM.

E.3.5 Vendor Information

The final wheel design, as depicted in Figure E.3-B, was fabricated by Advanced Urethane Solutions, Inc. The wheel was manufactured using Advanced Urethane Solutions 90A urethane. Advanced Urethane Solutions is located at 3912 Tyron Courthouse Road; Cherryville, NC. The contact for this project was Application Specialist Joe Bell.

E.4 Motor Selection (LW)

E.4.1 Determination of Required Torque

The torque required by the motor is that necessary to provide an acceleration of the flywheel in either direction that will counter the moments induced by the electromagnets. Due to the principle of the conservation of angular momentum, the torque required by the motor is equal to the torque resulting from the spin-up of two electromagnet dipoles. The greatest torque is the initial torque, the torque required to initiate the spin-up process.

The relationship for the magnetic moments of the (in this case) two dipoles is given in Equation E.4-1:

$$\mu_A \mu_B = \frac{32\pi m r^5 \sqrt{4\dot{\omega}^2 + \omega^4}}{3\mu_0}$$

Equation E.4-1

The relationship for α, the angle at which the magnet is rotated from its initial position, is given in Equation E.4-2:

$$\alpha = -\cos^{-1}\left(\frac{\omega^2}{\sqrt{4\dot{\omega}^2 + \omega^4}}\right)$$

Equation E.4-2

The relationship for the torque induced by the (in this case) two dipoles is given in Equation E.4-3:

$$\tau = \frac{-1}{4\pi}\frac{\mu_0\mu_A\mu_B}{(2r)^3}2\sin\alpha$$

Equation E.4-3

Combining Equations E.4-1, E.4-2, and E.4-3 yields a simplified expression for the torque induced by the dipoles:

$$\tau = \frac{-4}{3}\dot{\omega}\, m r^2$$

Equation E.4-4

      The boundary conditions applied to the system are given in Equation E.4-5. These conditions are derived from the requirements of the spin-up process. The dipoles begin spin-up from rest; therefore there should be no angular momentum in the initial state. The requirement for spin-up stipulates that the vehicle will be spinning at a rate of one revolution per minute at the conclusion of spin-up. This requirement drives the end boundary condition.

$$\omega(0) = 0$$
$$\omega(2\pi) = \frac{2\pi}{60}$$

Equation E.4-5

The equations given in Equation E.4-6 show the general relationships for ω and Θ (the angle between the dipoles in the x-y plane). This analysis assumes that acceleration is constant: $\dot{\omega}$ is constant.

$$\omega = \dot{\omega}t$$
$$\theta = \int \omega dt = \frac{1}{2}\dot{\omega}t^2$$

Equation E.4-6

From the given expressions above, including the boundary conditions, a value for $\dot{\omega}$ may be obtained:

$$\dot{\omega} = \frac{\pi}{3600}$$

Equation E.4-7

Applying a mass of 18 kg and a separation distance of 2 m to the system, the torque induced by the dipoles may be calculated. This torque is the torque required by the motor in order for the dipoles to spin up according to protocol.

$$\tau = 0.021\,\text{Nm}$$ **Equation E.4-8**

A margin of 375% was applied to this result for the motor selection process. The torque requirement for the motor was set at 0.1 Nm to insure that spin-up will occur properly, i.e. the system will function as specified.

E.4.2 Motor Comparisons

Two main classes of motors were examined: industrial brushless motors and model motors (for airplanes, boats or cars). The industrial brushless motors easily provide the

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torque required by the system. However, they are much more massive than the model motors and also more expensive by a factor of four. Most of the model motors are not capable of providing the necessary torque (and were therefore eliminated from the search). However, a certain class of model airplane motors is capable of providing sufficient torque for spin-up.

Both the brushless and model motors run from variable voltage supplies, but the brushless motors require a much higher range of voltage, ranging up to a high of 50 required volts. Such high required voltages place a large burden on the power system, also adding significant mass to the system through batteries. The voltage ranges for the smallest, most powerful model motors have high-end voltages of 18-24 volts, which is a more reasonable parameter for the EMFFORCE system.

E.4.3 Motor Selection

The selected motor is the Cobalt 625 Airplane Motor, provided by Astroflight, Inc. The motor requires a variable voltage of 6-18 volts. It is capable of providing a torque of 0.25 Nm with an applied current of 25 amps. This torque capability is more than sufficient for the EMFFORCE operational requirements. Noteworthy motor specifications are listed Table E.4-A:

Table E.4-A: Cobalt 625 Motor Specifications

Speed	971 RPM/Volt
Maximum Current	35 Amps
Shaft RPM	12,000 – 15,000
Weight	0.31 kg
Length	0.0635 m
Diameter	0.0427 m

E.4.4 Determination of Required Power

The power requirement for the motor depends on the voltage and current input to the system. The torque requirement for the motor drives the required current, while the voltage required is determined by operational speed of the motor. Equation E.4-9 gives the relationship for the motor's torque, where k is the torque constant.

$$\tau = ki$$
Equation E.4-9

The provided torque constant for the Cobalt 625 motor is $k = 1.39 \dfrac{in-oz}{A}$. This yields a required current for the motor of 10.2 amps.

The total voltage required by the motor is a sum of the voltage needed to overcome the resistance in the armature and the voltage needed to overcome back electromotive force (EMF):

$$V_{tot} = V_{BackEMF} + V_{arm}$$

Equation E.4-10

The voltage due to back EMF is calculated using a design RPM for the flywheel of 2000 RPM. This is a design requirement and has inherent margin built into it. Equation E.4-11 gives the relationship for the voltage due to back EMF:

$$V_{BackEMF} = \frac{\text{Design RPM}}{\text{Motor Speed Constant}}$$

Equation E.4-11

For the given motor and design RPM this gives a voltage due to back EMF of $V_{BackEMF} = 2.1\,\text{V}$.

The voltage needed to overcome the resistance in the armature is found using Ohm's Law:

$$V_{arm} = iR_{arm}$$

Equation E.4-12

The resistance in the armature is given to be 0.093 Ω. This gives a voltage due to resistance in the armature of $V_{arm} = 0.95\,\text{V}$.

These calculations give voltage, current, and power requirements for the system:

$$V_{tot} = 3.1\,\text{V}$$
$$i = 10.2\,\text{A}$$
$$P = iV = 32\,\text{W}$$

Equation E.4-13

E.4.5 Vendor Information

The reaction wheel assembly motors were purchased from Astroflight, Inc. which is located at 13311 Beach Avenue; Marina Del Rey, CA.