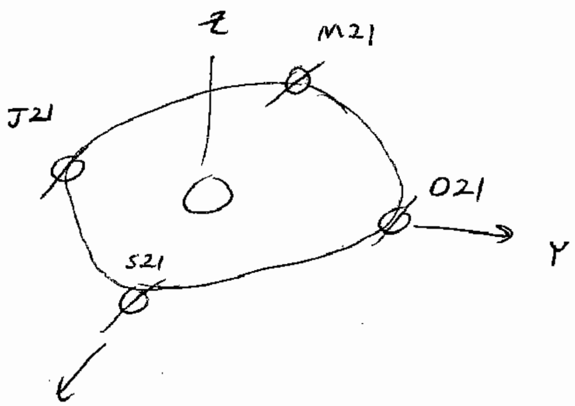
Coordinate Frames

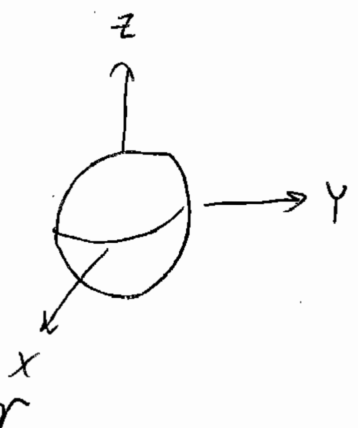
Heliocentric - ecliptic



- Useful for interplanetary transfer, since planets more or less in ecliptic.
- Nearly inertial, referenced to specific "Epoch".

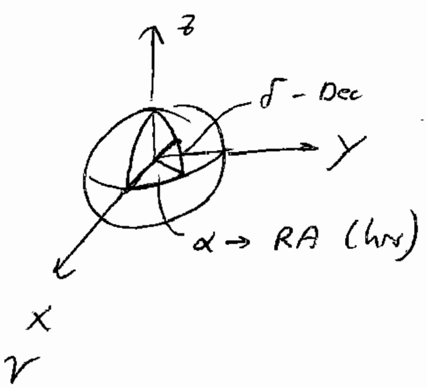
♈ X → - First point is Aries - intersection of Earth equitorial plane with Earth ecliptic (orbital plane) (now points to pisces)
- Defines point of zero right Ascension
- moves ~ 1° every 70 years

Geocentric - equatorial



- Intersects HE plane along equinox line.
- marks "celestial sphere"

RA - Dec



- Star catalogues

Classical orbital Elements

a - size of orbit (semi-major axis)

e - shape of orbit (eccentricity)

θ - position in orbit (True anomaly)

↳ Also M - mean anomaly

T - Time of periapsis passage

Ω - ~~###~~ longitude of ascending node

ω - Arg of periapsis

i - inclination

} orientation in space.

Alternates: $\Pi = \Omega + \omega$ longitude of periapsis (good if equatorial)

$u_0 = \omega + \theta$ Argument of lattitude at epoch (periapsis passage) (good if circular)

$$\left.\begin{aligned} l_0 &= \Omega + \omega + \theta \\ &= \Pi + \theta \\ &= \Omega + u_0 \end{aligned}\right\}$$ True longitude at epoch (good if equatorial and circular)

$\boxed{\text{Finding Elements}}$

$\vec{h} = \vec{r} \times \vec{v}$

$\vec{N} \equiv \hat{k} \times \vec{h}$ points to ascending node

$$\vec{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\vec{r} - (\vec{r}\cdot\vec{v})\vec{v}\right] \qquad e = |\vec{e}|$$

eccentricity vector → points toward periapsis

$$p = |\vec{h}|^2/\mu$$

$$\cos i = \frac{\vec{h}\cdot\hat{k}}{|\vec{h}|} \quad (i < \pi)$$

$$\cos\Omega = \frac{\vec{N}\cdot\hat{i}}{|\vec{N}|} \quad (\vec{N}\cdot\hat{j} > 0,\ \Omega < \pi)$$

$$\cos\omega = \frac{\vec{N}\cdot\vec{e}}{|\vec{N}||\vec{e}|} \quad (\vec{e}\cdot\hat{k} > 0,\ \omega < \pi)$$

$$\cos\theta = \frac{\vec{e}\cdot\vec{r}}{|\vec{e}||\vec{r}|} \quad (\vec{r}\cdot\vec{v} > 0,\ \theta < \pi)$$

$$\cos u_0 = \frac{\vec{N}\cdot\vec{r}}{|\vec{N}||\vec{r}|} \quad (\vec{r}\cdot\hat{k} > 0,\ u_0 < \pi)$$

J_2 effect

Geopotential Expansion (Assuming axial symmetry)

$$\phi = -\frac{\mu}{r}\left[1 - \sum_{k=2}^{\infty} J_k \left(\frac{r_{eq}}{r}\right)^k P_k(\sin L)\right]$$

Legendre Polynomials

not in book, but $F = -\nabla\phi = -\left(\frac{\partial}{\partial r}\left(-\frac{\mu}{r}\right)\right) = -\left(\frac{\mu}{r^2}\right) = \frac{-\mu}{r^2}$

more general form includes sectoral (longitudinal) and Tesseral (checkerboard) terms.

- cause periodic variations in orbital elements
- secular (increasing/decreasing over time) changes in Ω, ω

$$\dot{\Omega} = -\frac{3}{2} n J_2 \left(\frac{r_{eq}}{p}\right)^2 \cos i \quad \text{rad/sec}$$

$$\dot{\omega} = \frac{3}{4} n J_2 \left(\frac{r_{eq}}{p}\right)^2 (5\cos^2 i - 1) \quad \text{rad/sec}$$

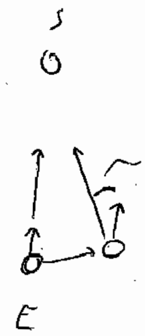
Special orbits

Geosynchronous - $P =$ ~~[illegible]~~ 1 day

How long is a day? What is a day?

solar day → 24 solar hours → ~~time [illegible] between meridian crossings~~

Time from one crossing of a meridian (by the sun) to the next crossing.



(365 days/year
360°/circle)

1 sidereal day
23.9345 solar hrs
(23 hrs 56 min 4.1 sec)
86164.1 mean solar seconds
= 24 sidereal hrs.
(86400 sidereal seconds)

1 solar day
24 solar hrs
24.0657 sidereal hrs
(24 hr 3 min 56.56 sec)
86636.6 mean sidereal seconds
(86400 solar seconds)

Remember:

1 solar day > 1 sidereal day
" hr " hr
" min " min

So... Geosynchronous $(e=0)\ (i\neq 0)$

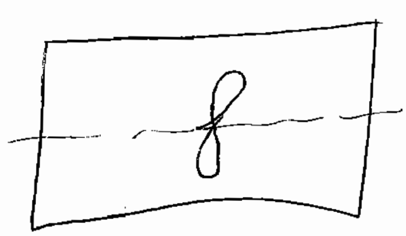
$P = 1$ sidereal day.

~~Altitude =~~ $P = 2\pi\sqrt{\frac{M}{a^3}}$

$a = 42{,}160$ km $\Rightarrow P = 86151.4$

NOT 86164.1
why? (J_2)

Ground Track



Geostationary $i=0$

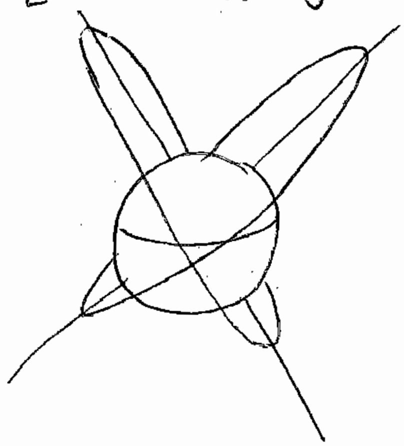
(N/S station keeping)

(i is perturbed by Sun and moon)

Frozen orbits (ex. Molniya)

$$\dot{\omega} = \frac{3}{4} n J_2 \left(\frac{r_{eq}}{p}\right)^2 (5\cos^2 i - 1) = 0$$

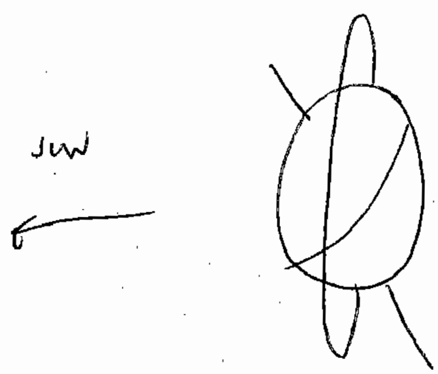
$$i = 63.4 \text{ deg}$$



- long dwell time in northern hemisphere
- line of apsides (greek) doesn't rotate (frozen)

Sun-synchronous -

To sun



$$\dot{\Omega} = + \frac{360}{365.25} = 0.986\,°/\text{day}$$

(solar)

$$= -9.96 \left(\frac{r_{eq}}{p}\right)^{3.5} \cos i \quad (\text{deg/day})$$

Take $e = 0$, $p = 7000$ km, $r_{eq} = 6378$ km

$$i = 97.88° \quad (i > 90, \dot{\Omega} > 0)$$

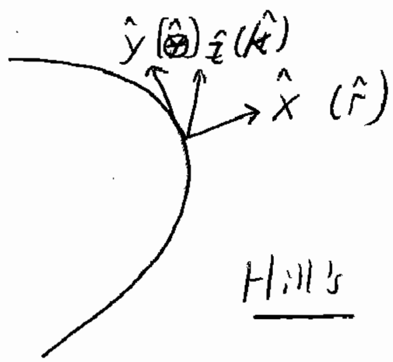
- Keeps out of Earth's shadow

cluster dynamics

What is the motion for 2 satellites with:

sat1	sat2	
$a = 7000\,km$	$a = 7000\,km$	→ same period
$e = 0.0001$	$e = 0.0001$	→ same shape
$i = 40°$	$i = 40°$	→
$\Omega = 0$	$\Omega = 0.0001°$	→ slight separation at equator
$\omega = 0$	$\omega = 180°$	→ perigee on opposite sides of equator
$\theta = 180$	$\theta = 0$	→ satellites in close proximity

Linearize $\ddot{\vec{r}} + \frac{\mu}{r^3}\vec{r} = 0$ around $r_0 = a$, $\dot{\theta}_0 = \sqrt{\frac{\mu}{r_0}}$



$$\vec{r} = (r_0 + x)\hat{r} + y\hat{\theta} + z\hat{k}$$

Hill's Equations

$$\left.\begin{aligned} \ddot{x} - 2\dot{y}N - 3N^2x &= 0 \\ \ddot{y} + 2\dot{x}N &= 0 \end{aligned}\right\} \text{coupled}$$

$$\ddot{z} + N^2 z = 0 \;\} \text{ uncoupled}$$

$$x = A_0 \cos(Nt + \alpha) + X_{offset}$$
$$y = -2A_0 \sin(Nt + \alpha) - \frac{3}{2} Nt\, X_{offset} + Y_{offset}$$
$$z = B_0 \cos(Nt + \beta)$$

Animation

What about with J_2 included? 9

modified Hill's Equations

Relative for ref orbit: Eq (23) pg 5.

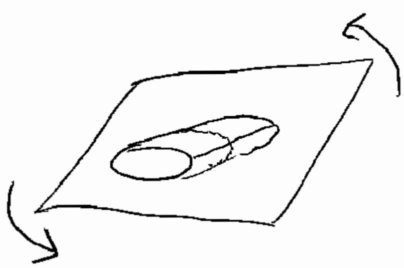
Relative To a ref satellite: Eq(41) pg 8.

$$\ddot{x} - 2(NC)\dot{y} - (5c^2-2)N^2 x = 0$$
$$\ddot{y} + 2(NC)\dot{x} = 0$$
$$\ddot{z} + k^2 z = 0$$

$$C = \sqrt{1+S}$$
$$S = \frac{3}{8} J_2 \left(\frac{r_{eq}^2}{r_{ref}^2}\right)(1+3\cos 2i_{ref})$$
$$K = N\sqrt{1+S} + \frac{3}{2} N J_2 \left(\frac{r_{eq}}{r_{ref}}\right)^2 \cos^2 i$$

Similar To Hill's, except

- period of coupled x/y motion is inclination dependent.
- period of z motion is also inclination dependent and distinct from x/y.



- still 2x1 ellipse
- still intersection of a plane
- plane now rotates around axis normal To 2x1 ellipse ($\hat{z}_{ref}$)

=> Projection on ground changes with Time.

[Animation]