MIT 16.90 Spring 2014: Problem Set 7

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Due: Monday April 14, in class

Problem 7. 1 *Method of Weighted Residuals*

Consider the 1D diffusion problem

$$\frac{\partial^2 T}{\partial x^2} = \sin(2\pi x), \qquad\qquad T(0) = T(1) = 1. \qquad\qquad (1)$$

1. Determine the analytic solution to Equation (1).

2. Assume the solution has the following form:

$$\tilde{T}(x) = 1 + a_1\phi_1 + a_2\phi_2 + a_3\phi_3,$$

   where $\phi_1 = x(1-x)$, $\phi_2 = x^2(1-x)$, $\phi_3 = x^3(1-x)$, and a_1, a_2, a_3 are unknown constants. Apply the method of weighted residuals using a Galerkin approximation and determine the 3×3 system of equations.

3. Can the 3×3 system of equations be solved to determine a unique solution (i.e., can you determine unique values of a_1, a_2, and a_3 that solve the Galerkin method of weighted residuals for this assumed solution)? If yes, give the values of the a_i. Create a plot that compares the weighted-residual solution and the exact solution.

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