### 10.52

## Mechanics of Fluids

Spring 2006
Problem Set 2

Problem 1 (Problem by James Fay, courtesy of MIT Press. Used with permission.)


A lock gate separates the water in the Charles River basin from the sea water in Boston harbor. As shown in the above figure, the water in the basin consists of an upper fresh water layer of density $\rho_{f}=1000 \mathrm{~kg} / \mathrm{m}^{3}$ and a depth of 4 m and a lower salt water layer of density $\rho_{\mathrm{s}}=1030$ $\mathrm{kg} / \mathrm{m}^{3}$. The fresh/salt water interface is 2 m above the sill of the gate. On the harbor side, the salt water depth $h$, measured above the level of the sill, rises and falls with the tide.
(a) At some water level $h$ on the harbor side, the total horizontal force of the sea water exerted on the lock gate will be balanced by the total horizontal force of the basin water acting against the other side of the gate. Calculate the value of $h$ at this equilibrium.
(b) Under this condition, if there is a leak in the lock gate seal at the sill, will water leak into or out of the basin?

## Problem 2 (Problem by Ain Sonin and Ascher Shapiro. Used with permission.)

A reservoir of water is closed at one end by a barrier. At the top of the barrier is an L-shaped channel that further restrains the water when its level is higher than the barrier top by the amount $h$, as shown in figure below. The channel is hinged at the corner point $O$, so that it can rotate in a counterclockwise direction, but not in a clockwise direction. If the water level $h$ is high enough, the pressure force on the vertical face of the L-shaped channel will maintain a clockwise moment that will exceed the counterclockwise moment of the pressure force on the horizontal leg of the channel, and the channel will remain upright and prevent water leaking beneath it. Calculate the minimum value of the ratio $h / H$ that will just ensure against rotation of the channel.


## Problem 3

One end of a 25 lb . wooden pole 3 in . in diameter and 18 ft . long is anchored with 10 ft . of light cable to the bottom of a fresh-water lake where the depth is 15 ft . Which of the two configurations shown describes the equilibrium position? Find the length $x$ of the pole protruding above the water. $\left[\rho_{\mathrm{H}_{2} \mathrm{O}}=62.4 \mathrm{lbs} / \mathrm{ft}^{3}, \mathrm{~g}=32.2 \mathrm{ft} / \mathrm{s}^{2}\right]$


Problem 4 (Problem by Ain Sonin and Ascher Shapiro. Used with permission.)
A cylindrical weir has a diameter of 3 m and a length of 6 m . Find the magnitude and direction of the resultant force acting on the weir from the water.


## Problem 5

A balloon filled with helium is attached to the floor of a stationary vehicle as shown in the figure. The vehicle now accelerates at a steady rate of $\underline{a}=0.25 \mathrm{~g} \underline{\mathrm{i}}$. Find the angle which the balloon string makes with the vertical at steady state. Sketch the position of the balloon relative to point A in the accelerating vehicle.


Problem 6 (Problem by Ain Sonin and Ascher Shapiro. Used with permission.)
The sketch shows a small airplane making a steady turn of 400 m radius while banking in a horizontal plane at $300 \mathrm{~km} / \mathrm{hr}$.

The fuel tanks are in the wings. Estimate the angle $\theta$ between the free surface of the fuel and the horizontal.


