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**PROFESSOR:** OK, let's just take 10 more seconds. OK. Does someone want to explain the answer here? I'm not sure if this is on. Give it a try.

**AUDIENCE:** So it says in the problem that the X-rays have the same wavelength, so you know that they also have the same frequency, so that discounts 1, 2, and 5 and 6. So then it's just a choice between 3 and 4. And in the video the other day, you said in order to image these proteins, you need high intensity light. So, 3.

**PROFESSOR:** Yup. That's a great explanation. Here, I don't really know what these-- these might come in handy today. I don't know. OK. Yeah, so the trick was better quality data, so you probably figured out that that, then, was the higher intensity. So this is a true thing. So we have data collection.

There's equipment here at home, and a lot of universities have what they call home data collection equipment, but we often travel to synchrotrons where we have higher intensity, i.e. more photons per second, and then you get better quality data. And so sometimes people do these things remotely where you ship your samples and someone else collects it, but my lab likes to go. And you stay up all night and collect great data, and it's a bonding experience. You saw a little bit of that on the video. OK.

We ended last time looking at the Schroedinger equation and seeing that the Schroedinger equation could be solved for a hydrogen atom, giving information about binding energy, the binding of the electron to its nucleus, and also a wave function, which we haven't talked about yet. So we're going to continue talking about this binding energy, and then next week we're going to move into wave functions, or orbitals.

So the binding energy that comes out of the Schroedinger equation, no one should ever just believe things. It looks fancy, but does it really work? Is it really doing this right estimation? And again, it just came out of Schroedinger's mind, so it's always nice to verify that this equation is working pretty well. So today we're going to talk about how we were able to verify

that the binding energy that the Schrodinger equation was predicting actually agrees with experiment. So we're going to continue talking about binding energies, then go on to the verification, with a demo, of how people were able to show that there was good agreement here.

All right. So let's continue with binding energies. So we're still talking about the hydrogen atom and energy levels, and we saw the last time that the Schrodinger equation could be derived for a hydrogen atom such that the binding energy, or  $e$  to the  $n$ , was equal to minus this Rydberg constant,  $R_H$ , over  $n$  squared where  $n$  is the principle quantum number. And so this is what we saw last time, and now we have a graphical depiction of this.

And you'll note that this is a negative value over here. So if  $n$  is 1 and we have the principle quantum number of one, we have minus  $R_H$  over one squared. And so we just have the negative value for the Rydberg constant,  $2.18 \times 10^{-18}$  joules. And as we go up here in energy, we would get to an energy of zero. And if energy here is zero, what must be true about  $n$ ? What kind of number is  $n$  here? Infinity, right. So if this is infinity, that number goes to zero.

And so if the electron is infinitely far away from the nucleus, it's basically a free electron. It doesn't feel any kind of attraction. It's infinitely far away. Then your binding energy would be zero, i.e. it's not bound, and that would be true with this infinitely far away distance. And then in between the  $n$  equals 1,  $2n$  equals infinity, we can use this equation for the hydrogen atom to figure out what these energy levels are.

So when we have the  $n$  equals 2 state, it would be minus  $R_H$  over 2 squared, or 4, and so we can calculate what that number is here, minus  $0.545 \times 10^{-18}$  joules.  $n$  equals 3 so we have  $R_H$  over 3 squared. We can do the math over here. 4-- you get the idea-- minus  $R_H$  over 4 squared, and we have then over 5 squared, over 6 squared, and you can see the energy, and you can calculate the energy levels here.

All right. So when you have an electron in this  $n$  equals 1 state, that's the lowest energy, it's the most negative number, and that's known as the ground state. And when you have an electron in this ground state, that's the most stable state for the hydrogen atom. So again, from these lower ground state up to this state here.

Now we're going to introduce another term which you'll hear a lot, and this is ionization energy. So the ionization energy, the amount of energy you need to put in to ionize an atom or release

an electron. So the ionization energy of a hydrogen atom in the  $n$ th state is going to be equal to the binding energy, but the signs of these are going to be different. So we have this equation where binding energy equals minus IE, the ionization energy.

So we talked about the fact that the binding energy is negative, and the ionization energy is always positive. So for the binding energy, when the binding energy is zero, it means the electron isn't bound. So a negative value for binding energy means that the electron's being held by the nucleus-- the electron's bound. For ionization energy, that's the energy you need to add to the system to release the electron, and you're always going to need to add some energy, so that's a positive number. So when you think about ionization energy, it's a positive number that you're going to be expecting there.

Now we can consider this same diagram, and we already talked about these energy levels and now we can think about these in terms of ionization energies as well. So the difference from this state, where energy is 0, to the ground state down here, the ionization energy-- the energy that's needed to ionize an electron that's in  $n$  equals 1 here, is going to be equal to minus the binding energy of that electron in that  $n$  equals one state.

So again here, it's not too hard if you know this information and this equation to figure out what the ionization energy is. So that's just, then, going to be the positive value of the binding energy. So binding energy minus Rydberg's constant here,  $2.18 \times 10^{-18}$  joules. So the ionization energy, then, for a hydrogen atom in the ground state is positive  $2.180 \times 10^{-18}$ . And I'm just going to try to use the same number of significant figures. I always try to pay attention to my significant figures.

All right. So we can do this again for the  $n$  equals 2 state, or the first excited state. So here's the  $n$  equals 2 state. So now we're going to be talking about this differential energy here. So the ionization energy for an electron in this first excited state. Again, that will be ionization energy equals minus the binding energy for that state, and so that's going to be, then, the positive value here. So the binding energy was minus, if we change this to-- this is  $18.5 \times 10^{-18}$ , or  $5.0 \times 10^{-19}$  joules. Try to keep the significant figures the same.

All right. So why don't you give this a try now and we'll have a clicker question. OK. 10 more seconds. A very long one second. OK. Interesting. So maybe you can talk to your neighbor and someone can tell me what the trick is here.

OK. We have someone who is going to tell us what the trick is.

**AUDIENCE:** The ground state is  $n$  equals one, and from there, the excited states go up one. So the first excited state is  $n$  equals 2, and then the third one will be  $n$  equals 4. And since you're looking for the ionization energy, you go to the energy for  $n$  equals 4, and you multiply by negative one, which is  $40.14 \times 10^{-18}$ th.

**PROFESSOR:** Great. Yeah, so let's see what I have. I need to get some more stuff. Sorry. OK. So the trick. It's not a hard problem. You just had to figure out what the third excited state meant, so once you've figured that out, it was pretty easy to get it right. OK. So keep that in mind. Now that you've potentially made that mistake, you will not make that one again.

All right. So now we can think about this also in more general terms-- only slightly more general terms, frankly-- which is to consider, for other one electron ions, we can have a more general equation. So we had, for the hydrogen atom, the binding energy,  $E_n$ , is minus Rydberg's constant,  $R_H$ , over  $n$  squared. And now I've added  $z$  squared, which is the atomic number. And for hydrogen, it's 1, so it wasn't around. We didn't need it before. But we can consider other ions that also have one electron. They will also work with this equation.

So there's a couple of things that kind of fall out of this. One, that an electron is going to be bound more weakly when  $n$  is a big number here, and so that makes sense from what we were looking at before. And that an electron is also going to be bound more tightly when  $z$  is big, and we haven't really talked about that because we've just been talking about the hydrogen atom, and so it always has the same  $z$ . But if you have a different  $z$ , you're going to have a bigger positively charged nucleus, and so it makes sense that you would then have a tighter bonding electron.

All right. So you might think, what are other things that have just one electron that this is going to apply to? And so far, of course, we've just been talking about our friend, hydrogen, that has its one electron and  $z$  equals 1. But we have ions that can also have one electron. So helium plus, and it has a  $z$  of 2, but when it's helium plus it only has one electron. Lithium, plus 2, also only has one electron, and it has a  $z$  of three.

And then what about something that's a one electron system with a plus 64. Without looking at your periodic table, what does the  $z$  have to be here? Yeah. So, 65. So in working these kinds of problems, if you're talking about a one electron ion or atom and it's not hydrogen, don't forget about  $z$ . And we need to have that in our equation.

All right. So talking about these binding energies now, out of the Schrodinger equation, you can calculate ionization energies if you know the binding energy. All of this is good, but how do we know that we can trust the Schrodinger equation, that equations really are working? So the way that they figured this out is from experiment, and particularly, experimentally figuring out what the energy levels were, and thinking, does this match with the Schrodinger equation?

So they were able to use photon emission to be able to do this. So let's consider what photon emission is, and then we're going to prove that this equation that I've been showing you actually holds. So photon emission, this is a situation that occurs when you have an electron going from a higher energy initial state going to a lower energy state.

And as it goes from this high energy state to the low energy state, there's a difference between these two energy states,  $\Delta E$ , and that's going to be equal to the higher energy initial state minus the energy in the final state. So there's this difference in energy between the two states, and the photon that gets emitted when this energy transition happens has the same energy as the difference between those. So the energy of the emitted photon is also  $\Delta E$ . So you emit all of that energy as you have that change.

So the difference here-- we can consider an actual case where we're going from an energy difference of  $n$  equals 6 to an energy level of  $n$  equals 2, and we can think about what the energy difference is between these two and we can just write that equation out. So the initial energy, the electron started at  $n$  equals 6, the energy of the  $n$  equals 6 state, and it goes to the energy of the  $n$  equals 2 state. So energy  $n$  equals 6 minus energy  $n$  equals 2.

All right. So of course if you know energy, you can know a lot of other things about the photon. So you can calculate frequency of that emitted photon. So again, we have our energy difference here and we can then solve for the frequency of the emitted photon, which is equal to the energy difference. That energy, divided by Planck's constant, and you could also write it out, the initial energy minus the final energy over  $h$ . All of these are equivalent things. And when you know frequency, we're talking about light here, so you can calculate the wavelength.

So let's think now about what we might expect in terms of frequencies and wavelengths, depending on the energy difference between the two different states. So here, if we think first about this electron with the purple line, we have a large energy difference here between this state and this state down here, between  $n$  equals 5 to  $n$  equals 1. So when we have a large

difference in energy, what do we expect about the frequency of the emitted photon? Is it going to be a high frequency or low frequency?

High. Yep. So large energy, high frequency. And so then what would be true about the wavelength of that emitted photon? Short. Right. Now if we had a small difference, say  $n$  equals 3 to  $n$  equals 1, which is a smaller difference in energy, what's true about the frequency here? Right. Low frequency. And wavelength? Right, long wavelength. Right.

So now we're actually going to see some photons being emitted, and let me just build in to this experiment a little bit. So we have an evacuated glass tube filled with hydrogen. And if you have negative and positive electrodes, you can emit light from this and then analyze the different wavelengths. So we are not going to be the first people to see this, but we're going to try this and we should observe these different wavelengths coming off. And after we observe them, we will try to calculate what they're due to, and then if the experimental results of the wavelength and frequency observed can be explained by the Schrodinger equation.

But first, let's actually see the visible spectra that is created by hydrogen. And so we have our demo TAs, and actually if all our TAs can help pass out some little glasses to help everyone see this. And when we're ready, we're going to do lights down. But let's get everything handed out first. All right. I got the lights.

**TA:** So this is a hydrogen lamp, and when you turn it on, the electricity excites all the hydrogen inside and then you see this glow from the electromagnetic radiation being emitted by these excited hydrogens relaxing down to the ground state.

**PROFESSOR:** Want to try the light?

**TA:** So we're going to try this for those of you who don't have the glasses, but let's see if this works. It was kind of there. Is that what we're supposed to see? You're supposed to see all of them. I guess you can't, really. I guess depending on how you move this thing, maybe you end up seeing all of them. Is it working?

**PROFESSOR:** It's not working.

**TA:** It's not working? No? Yeah, I know. It kind of works, depending on how I move this thing.

**PROFESSOR:** Sometimes it works really well. Should we just try walking? Can we hold it up and see whether people also can see it?

**TA:** All rights. So we're going to hold it up. See if you guys--

**PROFESSOR:** Can just see it without the camera.

**TA:** OK. So what you should be able to see, for those of you that have your glasses, is a continuous spectrum with the various colors.

**PROFESSOR:** You might have to get the angle right. Are people in the middle of the room able to see it? Can anyone see it? Yeah? People on the edge of the room, can you see it? I think it's harder from-- and the camera's blocking people a little bit.

**TA:** It works. I can see it.

**PROFESSOR:** Do you want to move it up farther, like in front of that? Try turning it around. All right. Maybe we'll turn it slightly, and then we can come down, maybe, and try it after class if it's not working very well. When it's tilted, are you having better luck over here?

All right. I guess we'll bring the lights back up, and I'll show you what you should have seen, if it didn't work for you. So how many people were able to see the spectra? OK. All right. So a good number of people. Great. I feel like this room is not as perfect for this as some other rooms. But there are some rooms that actually don't get dark at all, and then you can't really see anything. All right. So maybe if we have a chance, we can try again at the end.

All right. So this is what you should have seen. You should have seen these different series of lights, or series of colors coming off, and we're not the first people to see it. So J.J. Balmer, in 1885, reported seeing these colors, and he wanted to calculate the frequencies of the lights that you were seeing emitted from this. And so he did calculate the frequency, and then he tried to figure out the mathematical relationship between the different frequencies of light that he was observing.

And he found that the frequency equaled  $3.29 \times 10^{15}$  per second, times  $1/n^2 - 1/4$ , where  $n$  was either 3, 4, or 5. And he really didn't understand what the significance of this was, but it was pretty. You had hydrogen in this sealed tube, and there were colors that came off, and they had frequencies. So that's kind of where that stood for a little while.

So now let's think about what those different colored lights were due to. So we have here energy levels, and the transitions that we were observing are all going to be  $n_i = 2$  final

state. And we can think about why you didn't see any transitions to  $n$  equals one. Think about that. We'll come back to that in a minute. But there were these different transitions that were being observed from 3 to 2, 4 to 2, 5 to 2, and 6 to 2.

So now let's think about which colors-- which wavelengths-- are due to which of the transitions. So for the red one, what do you think? 3, 4, or 5 transitioned to 2? 3. It is 3. And you could think about that in terms of the smaller energy. That's the smallest energy, so that would be a low frequency and a long wavelength. So the one with the longest wavelength-- and red is the longest wavelength-- so that must be the transition from an initial  $n$  of 3 to  $n$  equal 2.

And then we can fill in the rest, so this one over here must have been  $n$  equals 4 to 2. This one here, then, would be the blue,  $n$  equals 5 to 2. And then the purple or indigo at the end,  $n$  equals 6 to  $n$  equals 2.

So they saw these four colors, there were these different transitions, and so then, now, we can calculate what the frequencies of these are and think about this, then, in terms of Schroedinger's equation and test Schroedinger's equation to see if it predicts this. So we can calculate the frequency, then, of the emitted photons, and we had frequency equals the initial energy minus the final energy state, or this  $\Delta E$ , over Planck's constant.

And from the Schroedinger equation, we know about what these energy levels are from Schroedinger. And this is, again, for hydrogen, so  $z$  equals 1, so  $z$  isn't shown. We have the binding energy equals minus  $RH$ , Rydberg constant over  $n$  squared. And now we can put these equations together. So we can substitute these energies in using these, and so we can do that here. We'll pull out Planck's constant, so  $1$  over  $h$ .

And then we can substitute in minus  $RH$  over the initial  $n$  level squared minus minus  $RH$  over the final  $n$  squared. And we can also simplify this a little more, pull out  $RH$  over here, and now we just have  $1$  over the final-- we have minus a minus, so we've rearranged this--  $1$  over  $n$  final squared minus  $1$  over  $n$  initial squared. And we have an equation that solves for the frequency in terms of Rydberg, Planck's constant, and what the principle quantum numbers are, what  $n$  is.

So let's look at this a little more. Now remember, this is all going to  $n$  final of 2, and so we can put that equation up here again. So when this is 2, 2 squared is 4. And if you remember back, Balmer had a 4-- had this part of the expression-- but he had a strange number over here that



he experimentally determined. But if you take  $RH$  and divide by Planck's constant, Rydberg constant divided by Planck's, you get that number that Balmer had found back in 1885--  $3.29 \times 10^{15}$  per second.

So when we plug in the values from Schroedinger's equations, you come up with the experimentally determined values for frequencies, or wavelength, of the emitted light. And of course, from the frequency, you can calculate the wavelength, and the wavelengths that were observed experimentally agreed with the wavelength you would calculate from the Schroedinger's equation to one part,  $n$  times  $10^{-8}$ . So the agreement was absolutely amazing.

So Schroedinger's equation, which was taking into account the wavelike properties of the electrons, were able to predict, for a hydrogen atom, what wavelengths you should see emitted in that hydrogen atom spectra. So this was really exciting. Schroedinger equation was working, we had a way of describing the behavior that we were observing for these electrons, and that was really incredible. And I think Balmer should get a lot of credit as well for all of this. And the people who were doing these early experiments, they didn't know what it was meaning, but they were coming up with the data that allowed to test theories later on.

OK. So this was a series going to a final  $n$  of 2, and so we have the Balmer series. That was the visible series that we were seeing. So what about, why wasn't anything going to  $n$  equals 1? And that's a clicker question. So at the end, I'm going to ask you to put up the winners. Is my number good? OK. 10 more seconds. OK.

So 71%. So the trick here is to think about-- well, actually, someone can tell me, maybe. What was the trick here to think about? I'll get a little exercise. I'll come up. How's everyone doing up here?

**AUDIENCE:**

So for the Lyman series, there's a more difference from the Balmer series, so there's more energy in the transition when it goes down back to the ground state. So for that, with more energy, it's going to be a shorter wavelength, and that's ultraviolet.

**PROFESSOR:**

So here it would be convenient to remember your orders of what are short and long wavelength kinds of light. OK. So we have the UV range, then. And so that's why you didn't observe it. It was happening, but you didn't see it because it was in the UV.

All right. So then we can go on and look at the other things that can happen here. So we can

have  $n$  final of 3, and you don't need to know the names of these series, but that would be near IR.  $n$  equals 4 would be in the IR range. So only some of what's happening is actually visible to us. We see beautiful colors from transitions, but there's other things happening, too, that are not visible to us.

So at this point, we're feeling pretty good about those energy levels, about the Schrodinger equation being able to successfully predict what kind of energy levels you have, that binding energy. And now, this verification was good, from your photon emission. But there's another property that you can have, which is photon absorption. So why don't we do yet another clicker question as a competition, after all, about absorption. OK, 10 seconds.

OK. So now we're talking about a different process. We were talking before about electrons. They're starting up in a higher energy level, going lower. But with photon absorption, we're going the other direction, so we're going from a lower state. We're being excited. They're absorbing energy and being excited, so we can have a final state that is higher, and the energy is gained in this process, so it's being excited.

All right. So we can think about the same things, then, in terms of absorption. So if we have a big energy difference, if it's absorbing a lot of energy, big energy difference, it's going to be absorbing light with a high frequency and a short wavelength. If there's a small energy difference, it'll absorb a photon with a low frequency or a long wavelength. And we'll come back to some of these ideas, actually, well into the course, and we'll actually look at some pretty colors.

So in this case now, we can calculate frequency again, but our equation is a little bit different. So we have the Rydberg constant and Planck's constant again, but now we have  $1$  over initial  $n$  squared minus final  $n$  squared, and so this term should be a positive term. We should be getting out a positive frequency. So if I take this again and put it up here, you want to think about whether, if you're talking about absorption or emission, that's what's telling you if the energy is being gained or lost.

So you're not going to have negative frequencies in one case. You're going to be absorbing the light of a particular frequency, or emitting light of a frequency. So pay attention to your equations, and think about whether your answer actually make sense when you do them. And again, all these equations are going to be provided to you in an equation sheet.

OK. So let's consider the summary of both of these things now. So we're talking about

admission versus absorption. And so we have this Rydberg formula, which is what this is called, and it can be used to calculate the frequency of either emitted photons or absorbed photons, so from either process. And if we want to make it more general, again, it's just a one electron case, but we can put our  $z$  in for any one electron ion.

So frequency equals  $Z^2$ , Rydberg constant over Planck's constant, and we have  $1$  over the final minus initial, or initial minus final, depending on which process you're talking about. And over here, we'd be talking, then, about emission. So our initial energy is higher going to lower, and when that happens, you're going to be releasing light with the energy difference that is due to the difference between these states. So we're going to have our electron is going to emit this energy.

In the absorption process for this equation, we're going to go from an initial state that's lower to a higher state. So we'll have a final state that's higher than initial. That's absorption. It's absorbing energy. It's getting excited, and so the electron is absorbing that energy.

So this really summarizes, now, what we need to know about binding energies. Schroedinger equation also tells us about wave functions, which is what we're moving into next. So that's all for today, except we have a very important announcement, which is congratulation to recitation 6. Lisa, you are the first winner of the clicker competition. Have a great weekend, everybody.