5.61 Fall 2017
Problem Set #1

1. *Transfer of momentum between a photon and a particle.*

A. Compute the momentum of one 500nm photon using $p_{\text{photon}} = E_{\text{photon}}/c$ where c is the speed of light, $c = 3 \times 10^8$ m/s, and $\nu = c/\lambda$.

B. You are going to use a photon to observe one point on the trajectory of a Na atom between a source and a target. Suppose the photon hits the Na atom and is permanently absorbed by the Na atom. What is the change in velocity of the Na atom?

C. Answer the same question for the photon hitting and being absorbed by an electron.

D. A photon of $\lambda = 500$ nm can determine the position of an atom to $\Delta x \approx 500$ nm. Compute $\Delta x \Delta p$ for detection of a Na atom by a 500 nm photon.

E. Suppose instead you use a 1 nm photon. Will $\Delta x \Delta p$ be smaller, larger, or the same as for a 500 nm photon?

2. **A.** A baseball has diameter = 7.4 cm. and a mass of 145g. Suppose the baseball is moving at v = 1nm/second. What is its de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

and will such a slow moving baseball diffract off of the stationary bat of a player attempting to bunt the ball?

B. How might you measure the velocity of a baseball moving at v ≈ 1nm/sec?

3. A pulsed Nd:YAG laser is found in many physical chemistry laboratories.

A. For a 2.00mJ pulse of laser light, how many photons are there at 1.06μm (the Nd:YAG fundamental), 537nm (the 2nd harmonic), and 266nm (the 4th harmonic)?

B. The duration of a typical Nd:YAG laser pulse is 6 nanoseconds. During the laser pulse, (2 mJ at 1.06 μm) what are:
(i) the number of photons/second, and
(ii) the power in Watts (Joules/second)?

4. **A.** *from McQuarrie, page 38, #19*
Given that the work function of chromium is 4.40 eV, calculate the kinetic

energy of electrons emitted from a clean chromium surface that is irradiated with ultraviolet radiation of wavelength 200 nm.

B. What are the speed and the de Broglie wavelength of the ejected electron from question **4A**?

5. *from McQuarrie, page 38, #21*

Some data for the kinetic energy of ejected electrons as a function of the wavelength of the incident radiation for the photoelectron effect for sodium metal are shown below:

λ/nm	100	200	300	400	500
KE/eV	10.1	3.94	1.88	0.842	0.222

Use some sort of plot of these data to determine values for h and ϕ.

6. **A.** *from McQuarrie, page 39, #32*

Derive the Bohr formula for $\tilde{\nu}$ (a modified form of Eq. 1.29) for one electron bound to a nucleus of atomic number Z.

B. Use the Bohr Theory to predict the wavelength (in Å) of the n = 2 ← n = 1 "Lyman α" transition of a U^{+91} atomic ion.

C. For the U^{+91} $n = 1$ Bohr orbit, what are the radius and the electron speed? Is there anything impossible about this result?

D. For U^{+91} $n = 1000$, what are the orbit-radius and speed?

Questions about complex numbers and complex functions of a real variable.

7. *from McQuarrie, page 49, #A-2:*

If $z = x + 2i\,y$, then find

(a) $\mathrm{Re}(z^*)$

(b) $\mathrm{Re}(z^2)$

(c) $\mathrm{Im}(z^2)$

(d) $\mathrm{Re}(zz^*)$

(e) $\mathrm{Im}(zz^*)$

8. *from McQuarrie,*

(a) page 49, #A-3:

Express the following complex numbers in the form $re^{i\theta}$:

(i) $6i$

(ii) $4-\sqrt{2}i$

(iii) $-1-2i$

(iv) $\pi + ei$

(b) page 49, #A-4

Express the following complex numbers in the form $x + iy$:

(i) $e^{\pi/4i}$

(ii) $6e^{2\pi i/3}$

(iii) $e^{-(\pi/4)i + \ln 2}$

(iv) $e^{-2\pi i} + e^{4\pi i}$

9. *from McQuarrie, page 49,50 #A-6 – A-8 and A-10*

(a) Show that

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

and that

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}.$$

(b) Use McQuarrie **A.6** Equation to derive

$$z^n = r^n(\cos\theta + i\sin\theta)^n = r^n(\cos n\theta + i\sin n\theta)$$

and from this, the formula of de Moivre:

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta.$$

(c) Use the formula of de Moivre, which is given in part **(b)**, to derive the following very useful trigonometric identities

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$$

$$= 4\cos^3\theta - 3\cos\theta$$

$$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$$

$$= 3\sin\theta - 4\sin^3\theta$$

10. *from McQuarrie, page 50, #A-9*

Consider the set of functions

$$\Phi_m(\phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi} \qquad \begin{cases} m = 0, \pm 1, \pm 2, \dots \\ 0 \le \phi \le 2\pi \end{cases}$$

First show that

$$\int_0^{2\pi} d\phi \Phi_m(\phi) = \begin{cases} 0 & \text{for all values of } m \neq 0 \\ \sqrt{2\pi} & m = 0 \end{cases}$$

Next show that

$$\int_0^{2\pi} d\phi \Phi_m^*(\phi) \Phi_n(\phi) = \begin{cases} 0 & m \neq n \\ 1 & m = n. \end{cases}$$

======================== **Optional Problem** ==========================

11. (*from Karplus and Porter, page 37, #1.14*)

A. The force laws for electrostatic and gravitational attraction are identical. From handbook values of the masses of the earth and moon, the mean distance between them, and the gravitation constant *G*, calculate the value of *n* for the Bohr model of the earth-moon "atom". Is this result meaningful? Explain. [For data, see "earth" and "solar system" in *Handbook of Chemistry and Physics* (The Chemical Rubber Company).]

B. A frontier area in molecular spectroscopy is the study of "heavy Rydberg" systems where an atomic anion like F^- orbits around an atomic cation like Na^+. Compute *n* for a heavy Rydberg system of $Na^+ F^-$ with a separation of exactly 10 Angstroms. Also calculate the *n*, *n*+1 energy spacing (it will be REALLY small). For this problem, where the masses of the two particles are similar, use $\mu = \frac{m_a m_b}{m_a + m_b}$ as the mass in the Bohr atom formula.

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