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BOB FIELD:

I'm Bob Field. This is my 44th year at MIT, and I've taught this course roughly half the time, so it is my favorite course. And I'm a spectroscopist, and that means that I use quantum mechanics almost like breathing. And I'm going to try to convey some of the beauty and utility of quantum mechanics.

This is not a typical undergraduate quantum mechanics class. It's not about history. It's not about philosophy. It's about use-- use for understanding complicated stuff, use for insight. And so a lot of the material is not in the assigned text, which is a very good book, but it's mostly a safety net.

The printed notes, all of which are posted except for a few lectures that I'm still working on-- and that is the text. Everything in the notes, you're responsible for. There are sections of the notes which I call non-lecture. That's explanation of what's in the lecture or a little bit going beyond it. You're responsible for that. If I don't finish all the material in the notes, you're responsible for the material I didn't finish. You can ask me questions about it. You can ask your TAs questions about it.

But there's a supplementary text, which is a book that I wrote which is intended to take undergraduates like you to the frontier of research in spectroscopy. This is the book that I used to use to teach graduate quantum mechanics.

So I've written a lot of stuff which is already available on the web, but I just want to make sure that you know the structure of the course. There are going to be nine problem sets, always in weeks there is not an exam-- when there is not an exam. And the problem sets are worth 100 points.

There will be three 50-minute exams, to which you will be allotted 90 minutes. And they'll be evening exams and on Thursday nights. OK, there will be a final during final exam period. And so the points for the course add up to 600.

For each exam, you will be entitled to bring one 8 1/2 by 11 page that you've prepared with as

dense or as coarse writing as you want-- a different one page, only one page, for each exam and for the final. One page-- the exam-- one page for the exam. One page for the final. Not four pages for the final, not two pages for the second exam, et cetera-- one page. I believe that writing these pages of notes or pages of things you should remember provide structure that you actually learn by preparing this, and so I stick to this.

My job here is to force you to suspend what you expect about the way matter and light behave. You expect things based on your experience with macroscopic objects, and you're very smart. You're here at MIT because you are able to integrate those expectations and to express them in mathematical analysis of your observations. So you're pretty good at that.

But quantum mechanics forces you to step away from what you think you understand perfectly because it doesn't apply in microscopic systems. And so I'm going to destroy your expectations about how things work at the beginning, and by the end, I will give them back to you in the form of things called wave packets. Wave packets are quantum mechanical particle-like objects. They obey the rules, and they do a lot of the things you expect. But initially, you have to suspend that belief in particles.

OK, so I'm going to start by throwing a piece of chalk. I have to throw it at my TAs. So here is a piece of chalk, and the chalk followed a trajectory. If you're a major league outfielder, you look at the initial part of a trajectory, and you can pretty much figure out where you have to go.

This sort of thing would be the subject of 801, but the outfielders don't know physics. They just have instincts, and they know that if they look at the beginning of a trajectory, they can predict where it will end and when it will end, and this is really important. That's the macroscopic view. We talk about trajectories.

In quantum mechanics, there are no trajectories. We have possibly an observation of what was the initial condition and what was the detection event, but we can't describe what's going on by observing, point by point, the position and momentum. We're only allowed to do what we call click-click experiments. We start something in some kind of a well-prepared state, and then we detect what has happened at the end. We might do something to the system in between, but we cannot observe everything as the system evolves.

And here, I will attempt to a little bit justify that you need to suspend what you believe. I threw a piece of chalk. It could have been a baseball. And suppose now I said, let's decrease the

mass of the thrower, the catcher, and the object by 100. You pretty much know exactly what will happen.

Now let's decrease it by a factor of 10 to the 20, which is like going from a baseball to an electron. You think you might know, but I guarantee you don't know. And you're going to be surprised, and it's because in the microscopic world, observation modifies the state of the system, and any sort of interaction of the evolving system is going to affect what you observe.

And so we have to create a formal structure, which is a kind of measurement theory. What measurements are we allowed to make, and what do they tell us? Because you cannot observe the time dependence. You can calculate what the time dependence is if you know enough, but you cannot observe the thing evolving except by destroying it.

OK, so quantum mechanics is beautiful because it describes the microscopic world, and it doesn't tell you you're wrong about the macroscopic world. It matches everything the macroscopic world does.

OK, some of the key ideas of quantum mechanics-- so you do a series of identical experiments. You don't get the same result. It's probabilistic, and that should bother you because if you do an experiment carefully, you're trained to think that you do it carefully, you'll get the same result. But quantum mechanics says, tough luck. You can't do an experiment that carefully.

There's also this wave-particle duality. You know what particles are and what the properties of particles are. You learn that in 8.01. You know what waves are, and you probably also learned that in 8.01 or 8.02. But you know instinctively that particles and waves behave differently. But in quantum mechanics, everything is both particle-like and wave-like, and this is also something that should bother you.

And the third thing is energy quantization. Now, I said I'm a spectroscopist, so I live and die by these spectra, which consist of transitions between energy levels. And these transitions between energy levels encode everything we want to know about the mechanics of an object-- its structure, what it does.

And so quantum mechanics is an elaborate encoder of information, and it's usually encoded in the form of these quantized energy levels. And so you're going to want to be able to calculate how the energy levels or the spectrum that you would observe for an object is related to the

thing that describes what it can do. And that's the Hamiltonian, and the Hamiltonian we'll look at in many useful ways.

Now, I just want to warn you that there are two ways of presenting quantum mechanics-- the Schrodinger picture, which is differential equations and wave functions, and the Heisenberg picture, which is matrix mechanics, where we have matrices and we have eigenvectors.

And I am an advocate. I'm passionate about the matrix picture because what it does is presents everything in a way that you can then organize your insights, whereas the Schrodinger picture mostly involves solving one complicated differential equation after another, and the focus is on the mathematics, and it's much harder to see the big picture.

Now, that means you need to know a little bit of linear algebra. Now, most of you haven't taken a course in linear algebra, but that doesn't matter because the amount of linear algebra you need to know for quantum mechanics is extremely small, and I will probably present all of it in lectures. But you will definitely have the TAs as a resource, and it's possible that they will give some formal lectures on linear algebra or some handouts, but it's not complicated. It's beautiful.

OK, so light. Light is electromagnetic radiation. You've heard that. And it is both wave-like and particle-like. Now, the particle-like aspect of light is going to bother you, but it's very easy to show that it's necessary.

OK, so first of all, what are some wave characteristics? If you have a wave, what kind of measurements are you going to make on it? OK, Sasha.

AUDIENCE: Intensity at a point in space.

BOB FIELD: There can be intensity at a point in space, but there could also be particles that are impinging on that point, so you need something a little bit more that is definitely wave-like, and you have--
- yes?

AUDIENCE: It has a frequency and a wavelength and an amplitude.

BOB FIELD: Yes. I'm angling for something more, but it does have a frequency and a wavelength, and wavelengths are how you understand interference effects. And when you put light through a lens, there's refraction. When you put light on a grating, there's diffraction-- or through a pinhole. And there's the two-slit experiment, where you send light on an object which has two

slits, and you can't tell which slit the light went through. This is a very beautiful thing which I will talk about in a coming lecture-- in fact, lecture number three. So the key properties of waves are refraction, diffraction, two-slit experiment. And behind that is interference effects.

So we can have just a comic book picture or a cartoon. So here is a wave, and here is another wave. And this other wave has exactly the same frequency and phase. And if we add these two guys together, we get something that looks like that. And in fact, the addition is a little bit nonlinear.

So we have constructive interference, or we can have destructive interference and anything in between. If we add these two guys, what you get is nothing. So here, you get an intensified wave-- constructive. And here, we get cancellation.

Now, it's a little bit stressful to think that if particles have wave characteristics, they can annihilate each other, so you'll have to be prepared for that. So quantum mechanics exploits constructive and destructive interference. That's at the core, and you have to get used to that, and you have to get used to seeing particles do that.

OK, so waves have a frequency and a velocity and a wavelength. And now here, what is ν ? ν is c over λ . If someone on the telephone or on Skype constantly asks you "what is ν ," you can put an end to that behavior by saying c over λ . And if you do it often enough, it'll stop. It may be that the question will be rephrased, but it's useful to remember this as a repellent and also as a crucial organizing principle.

OK, waves are electromagnetic, and so that means we have transverse electromagnetic waves. And so here is one part of the wave, and here is another part of the wave. And this is the electric part of the wave. And so we have zero, and here is E_0 . And this is the magnetic part of the wave, zero and B_0 .

And this is in the xz plane. This is the z direction. This is the z direction. And this is the yz plane. Now, this corresponds to a wave which is linearly polarized along the x -axis.

And so this is-- it's kind of hard to draw a transverse electromagnetic wave, but there's an electric part and a magnetic part. And for the most part, I forget about this. The magnetic resonance-- this is the only thing you care about, but they're both part of light. And OK.

So we have these waves, and now, the intensity of the light is proportional to the E_0 squared-- the electric field squared. And it's also measured in watts per square centimeter. And this is in-

- so this is in volts per centimeter squared.

Now, I'm an old fashioned guy. I use centimeters instead of meters. You'll just have to forgive me for that, not using MKS units. But the important thing is that the intensity is related to the square of an electric field.

So suppose we have light impinging on a hunk of metal. So a metal is something where the electrons are free to move around, and so we like a metal because of that. We could ask, well, what happens if we put light on salt or on some organic molecule?

And in some sense, there is going to be a relationship, but the easy thing is if we put a beam of light onto a metal and it's pushing the electrons around. That's what an electric field does. It moves the electrons. And so you would expect classically that at high enough intensity, you're going to start ripping electrons out of the metal, and that would be wrong. And this is the photoelectric effect.

This is what we observe. 0 -- the current divided by the charge of the electron. So that's the number of electrons per second, and this is the intensity. And if we have infrared radiation, nothing happens, no matter how strong it is.

If we have ultraviolet radiation, we have something happening. As the intensity of the light increases, the number of electrons per second ejected from the metal increases linearly. So why is that? We expect that it would be intensity that determines it, but it's the color of the light that determines.

We can also do something like this, where we again ask, what is the current divided by the charge of the electron versus frequency? And again, we have 0 here. And what we see is nothing up until some critical point, and then this is-- so this is some special frequency, which is different for every metal. And then, all of a sudden, we start getting electrons. And this increases linearly, or at least for a while increases linearly with the frequency.

So there's something about the frequency of light that rips out the electrons. It has to be above a certain minimum, and then the electrons increase with the frequency, but in a little bit more complicated way than linearly. And so I have something in the notes which is not quite right, but this onset is important.

So these observations suggest that the electron is bound to the metal by some energy which

we call the work function, and it's called ϕ . And so this is the energy that it takes to rip an electron out of a metal. And so it's called the work function, and work functions for metals range from a little over 1 electron volt to around 5 electron volts.

I'm going to use those units. Well, you can forget that. But all metals are within a relatively narrow range of work functions, and this is somehow related to ν_0 . So this is an energy, and this is a frequency. And so we expect that there is some relationship between ν_0 and this energy, and there is some proportionality concept, which I can call anything I want, but I'm going to call it h because it's going to become Planck's constant.

That's the proportionality concept. And so the onset of ejection of electrons is when the frequency of light is greater than the work function. As I said, every metal has a different work function.

OK, so this is looking like somehow, the light does not act in an additive way on the metal. It acts in a singular way somehow. There are particles of light that have definite energy. This is what it looks like, and we're going to call those particles of light photons. And so now we're trying to think of, all right, electromagnetic radiation comes as particles. What are the properties of particles?

Well, particles-- well, I'm getting ahead of myself, but as long as I said this, particles have kinetic energy and momentum. Kinetic energy is a scalar quantity. Momentum is a vector quantity. And right now, what we want to do is measure the kinetic energy of the electrons that are produced by the annihilation of photons.

OK, and so we can imagine an apparatus like this. Here is our metal, and here is the light impinging on the metal. And we have grids. We have a ground, and we have another grid and a detector.

So we have ground voltage is 0. Here we have a voltage less than 0. Electrons don't like that. But these two grids are at the same potential. So the electrons don't know which way to go. And then there's another grid where we have the potential is V plus V stop. And so what happens is if the electron is ejected with enough kinetic energy to go uphill and cross through this potential, this grid, then it will make it to the detector and we'll count it.

So this is just a crude apparatus. If you were going to do this experiment, you would probably design it better. But the idea is what we want to do is to measure what is the voltage that we

apply that causes the electrons to stop reaching the detector. And that's the way we measure the kinetic energy of the electrons. When all of the electrons stop hitting the detector, we know the voltage, the stopping voltage.

And we can now plot the stopping voltage, V_{stop} , versus frequency. And here is ν_0 . And what we see is this. We see that below ν_0 , there are no electrons ejected. Once we're above ν_0 , electrons start being ejected. And they have a kinetic energy, which is measured by V_{stop} . And it increases linearly with frequency.

Now, when we do this experiment on different metals, the slope is constant, the same. It's universal. So the kinetic energy of the ejected electrons is equal to some constant. It goes as $h\nu - h\nu_0$. And this is the same constant that we saw before. And this is Planck's constant. And now what we've seen is that these particles of light have definite energy. They have definite kinetic energy. And we can stop and they transfer that energy.

And so we can draw an energy level diagram. So here's 0 and here. So this is energy. And this is minus the work function. And so we have a photon, which has this energy. And we have the kinetic energy of the electrons. And this is $h\nu - h\nu_0$. OK. This story is complete. The photoelectron effect is the easiest thing to understand at the beginning of quantum mechanics. And it says that light comes in particles, which we call photons. And the energy of a photon is $h\nu$. h is a fundamental constant. And we know what ν is.

OK. I'm going to talk about Compton scattering. But Compton scattering is a little bit harder to understand than the photoelectron effect. And I'm going to put equations on the board which I'm not going to derive. They're easily derived. But this is just to complete the picture of the particle-like characteristic of Planck.

So for Compton scattering, we have a beam of x-rays. X-ray is a form of light. It's a very high energy form of light. We have a block of paraffin. And what we observe is there are electrons kicked out. And there is x-ray scattered. So the experiment is we're looking for the particle characteristics. Particles have kinetic energy and they have vector momentum. And you have been trained painfully and completely in conservation of energy and momentum, because it enables you to solve all sorts of useful problems.

So suppose the x-ray comes in, hits the block. And so we have the incident momentum and the x-ray is backscattered. And the scattering angle θ is measured this way. So the difference in momentum for this scattering is large for backscattering and much less for

forward scattering. Now, this momentum has to be transferred to the electron. And so we can draw conservation diagrams. Or for this case, the backscattering. We have p_n . We have the electron, p electron. And we have the momentum of the x-ray.

OK, so this diagram determines the momentum transferred to the electron. And if the x-ray photon transfers energy to the-- it transfers momentum to the electron, it also transfers energy. So what we're going to see-- and I'm just going to wave my hands, because the mathematical analysis is something you can do, but I don't want to go through it. I just want to show the structure of the argument.

What you measure is the wavelength of the x-ray out minus the wavelength of the x-ray in. And so if the x-ray has transferred energy to the electron, it has less energy when it leaves, and it has longer wavelength. So this is measurable, the change in the wavelength of the x-ray, and that's going to be-- so we'll call that $\Delta\lambda$. And that's going to be a function of the scattering angle.

So Compton scattering basically says, OK, light is a particle. Particles have kinetic energy and momentum. Conservation of energy and momentum predicts a difference, a redshift of the photon, which depends on the scattering angle. The rest is all 8.01. You can do that. OK. And so what you end up finding is that if the scattering is 0, then $\Delta\lambda$ is 0.

There is no momentum. The photon just goes through. If the scattering is π , in other words, it's perfectly backscattered, then $\Delta\lambda$ you can calculate is $2h$ over the mass of the electron times the speed of light. Now, this quantity, h over the mass of the electron times the speed of light, has dimensions of length. And it's 0.02 or 3 angstroms.

And this is called the Compton wavelength of the electron. Because the photon is scattering an electron. The change in wavelength of the photon is determined by this. This is the momentum transfer. This gives you the momentum transferred. And so this is a universal constant. It says electrons have, when they're scattered out of any material, have this behavior. Again, universal and strange.

The actual experiment, since what you actually want to measure is the change in λ over λ . The change in λ doesn't depend on λ but this does. And so what you want to do is make the observable thing large. And so you go to a short wavelength to make this a large fractional change that's easily measured.

OK. The equations are derived in the lecture notes and better in texts. And what I want to do now is just tell you where we're going. We've talked about the wave and particle nature of photons, of light. And the particle nature is a surprise, but it's perfectly understandable and observable.

The next thing we want to do is determine the wave nature of things that we call particles. And so we're going to do other kinds of simple scattering experiments where we discover that the electron-- so we have some solid. And we have UV light or x-rays. And we go through this solid.

And it's mostly transparent. It's mostly free space. And so this light is interacting with the electrons. And maybe with the other stuff that is involved in matter, the nuclei. But the important thing is that the electrons that get scattered are in a kind of a diffuse state. This is mostly nothing. This material doesn't scatter x-rays. It's mostly transparent.

And so an explanation for that is the Rutherford planetary model, where we have a nucleus with a charge and electrons orbiting that nucleus. And that's all very nice. It's a way of saying, well, OK, the electrons are forced to choose distances from the nucleus. And there's mostly empty space. But then you look more closely, and the electrons are doing this. They're oscillating in space, and they're going to radiate energy. And this radiation of energy will cause the electrons to spiral in and combine with the charge in the middle. And that's a problem.

And so we have an explanation for why matter is mostly empty space and a conundrum. The electron should recombine with the nucleus. So how can matter be relatively non-compressible? And the best explanation is this planetary idea. But then we have this thing we have to explain.

Now, there's two hypotheses for this, which are really just ad hoc things. One is the Bohr model where it says, in order for the electron to obey some laws of physics as it orbits the nucleus, it conserves angular momentum. Well, if it were rotating combined with the nucleus, it wouldn't do that. And so that's sort of OK.

And then there's de Broglie, who was a very smart person. And he said that in order for the electron not to annihilate itself as it goes around the orbit, it has to have an integer number of half wavelengths along that trajectory. Then it won't annihilate itself. And that is much better than Bohr's hypothesis. But both of these hypotheses lead to line spectra hydrogen where you can build a really simple model and you can predict to eight decimal places all the absorption

transitions of hydrogen atom. So a very simple model leads to an incredibly powerful and mysterious result.

There is another experiment that I will talk about next lecture. And that is suppose we have a thin sheet of metal. We have photons, x-rays, or UV photons, impinging on this metal. There's diffraction because there is regular distances between the atoms in the particle in the foil.

And so what we measure is diffraction rings for the electron and for the photon. And the diffraction rings are identical when you choose the wavelength of the particle to be equal to the wavelength of the photon. And so this is another way of showing wave particle duality. We can show that they have wavelengths and we can calculate to make them the same.

So this is all very exciting. And this is about as much philosophy and history as you're going to get from me. Once we understand that there is something that we have to do, which is called quantum mechanics, we're going to start solving problems and then being able to understand incredibly complicated effects beyond the simple problems.

And that should be exciting, but it should also be a little bit disturbing, because I will use a technique called perturbation theory, which for some reason all of the textbooks for a course at this level either ignore or treat in the most superficial way. But perturbation theory is a way of taking problems that we understand and can solve exactly. And exactly means an infinite number of states, all of which are given to us by solving one equation. And we can then use them to understand problems we can't solve exactly. And there'll be several ways in which I deal with that.

OK, so it's time to stop. And I hope you're excited about what lies ahead, because it is strange and wonderful. And it says we can look inside a molecule. We can make measurements. We can understand what this molecule is going to do. But we have to develop our new way of doing that. OK, thank you.