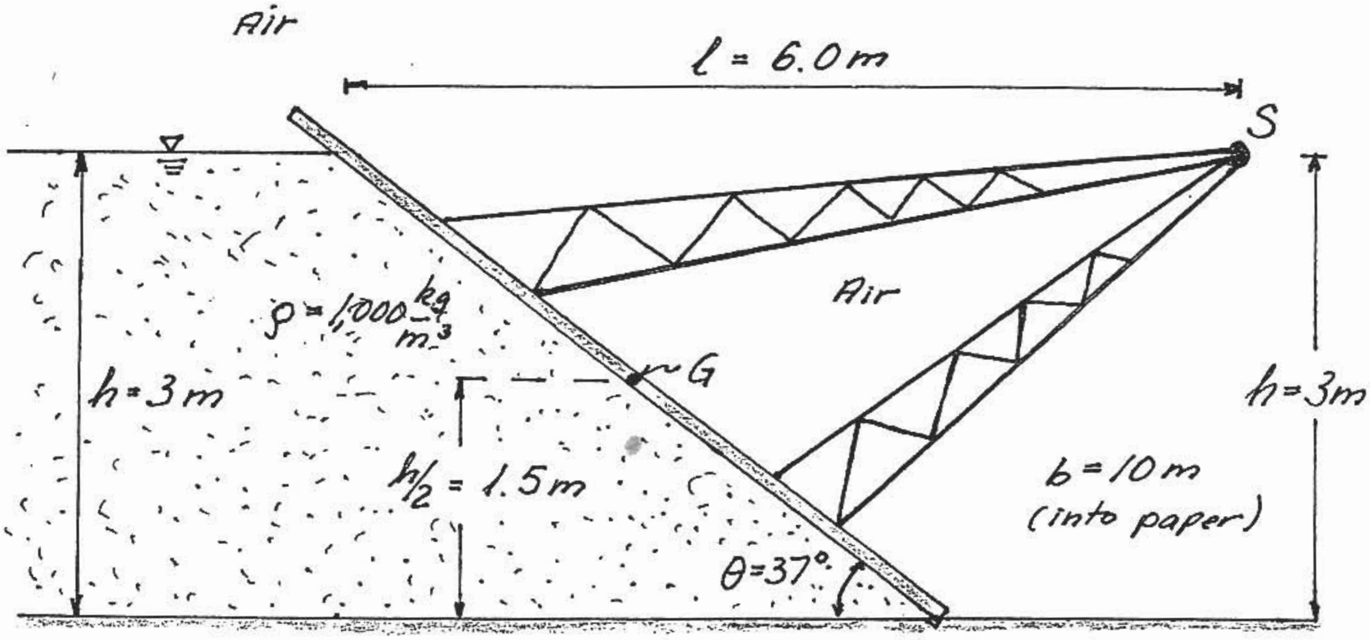
Massachusetts Institute of Technology
Department of Civil and Environmental Engineering

1.060 Fluid Mechanics
In-class Examination
March 9, 2001

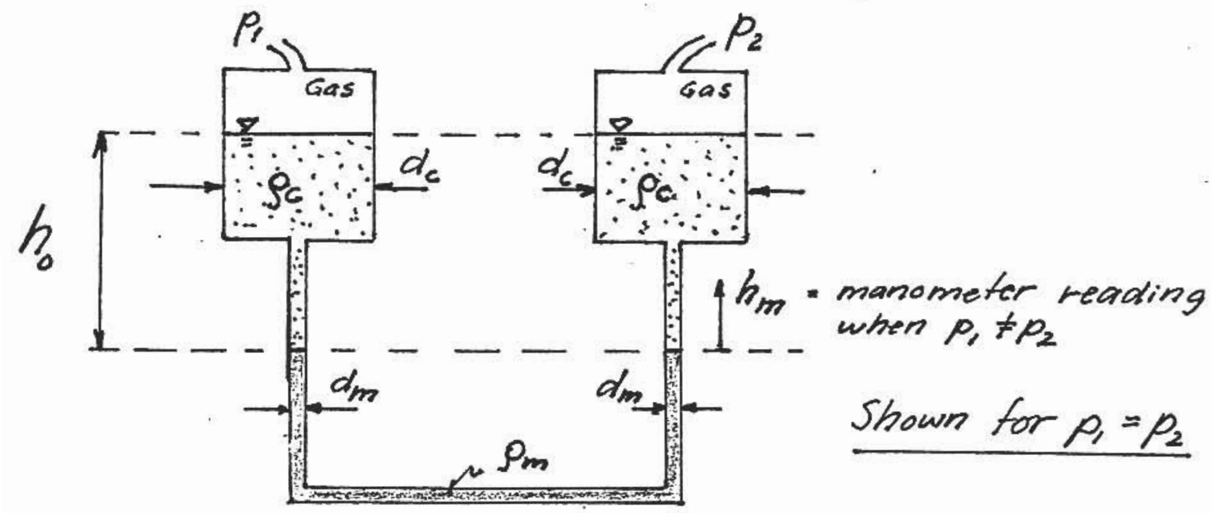
Problem 1 (35%)



The sketch above shows a plane gate that backs up water (ρ = 1000 kg/m^3) in a 10-m-wide rectangular channel to a depth of h = 3.0 m. The gate itself is inclined at an angle of θ = 37° to horizontal and is supported by a shaft, S, spanning the width of the channel. The shaft is located h = 3.0 m above the bottom at a distance l = 6.0 m "downstream" of the gate's intersection with the still water level in the channel. The total weight of the gate is W_g and is assumed to act through the mid-point of the submerged portion of the gate (point G in sketch).

a) Determine the total horizontal pressure force acting on the gate, F_H, and its line of action.
b) Determine the total vertical pressure force acting on the gate, F_V, and its line of action.
c) Determine the necessary weight of the gate, W_g, for the gate to remain closed.
d) Corresponding to the weight of the gate obtained in (c) determine the total force acting on the shaft, S.

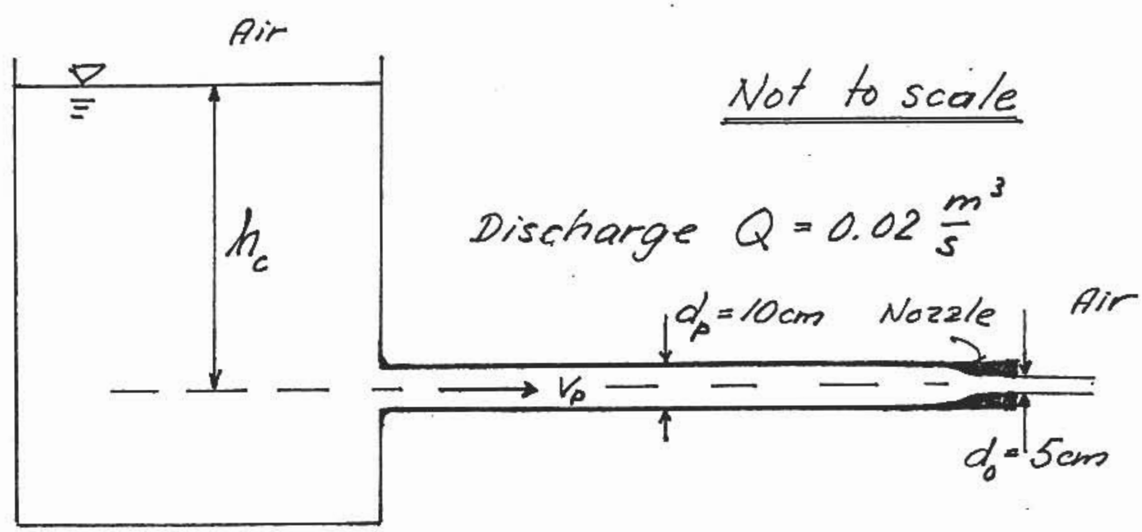
Problem 2 (35%)



The sketch above shows a precision manometer used to measure small pressure differences in gaseous systems. It consists of two circular cylindrical chambers, of diameters d_c, partially filled with a chamber fluid, of density ρ_c. The bottoms of the two chambers are connected by manometer tubes, of diameter d_m, containing a manometer fluid, of density ρ_m. When the two chambers are open to the atmosphere, i.e. when $p_1 = p_2$, the manometer is in equilibrium as shown in the sketch. When one chamber is connected to a pressure p_1 and the other to a pressure p_2 (in a gas) the manometer fluid in the right leg (connected to the chamber with pressure p_2) is "deflected" a vertical distance, h_m, from its equilibrium position.

a) Derive the general expression for the pressure difference p_1-p_2 corresponding to a manometer reading of h_m.
b) Evaluate the expression obtained in (a) for $d_c = 10$ cm, $d_m = 1$ cm, $\rho_c = 970$ kg/m^3, $\rho_m = 1000$ kg/m^3, and $h_m = 5.0$ cm.

Problem 3 (30%)



The sketch above shows a horizontal pipe of diameter $d_p = 10$ cm, carrying a discharge $Q = 0.02$ m^3/s of water, $\rho = 1000$ kg/m^3. The discharge is supplied from a very large container in which the water level is a vertical distance h_c above the pipe elevation. The pipe discharges into the atmosphere through a nozzle with a circular outflow of diameter $d_o = 5$ cm.

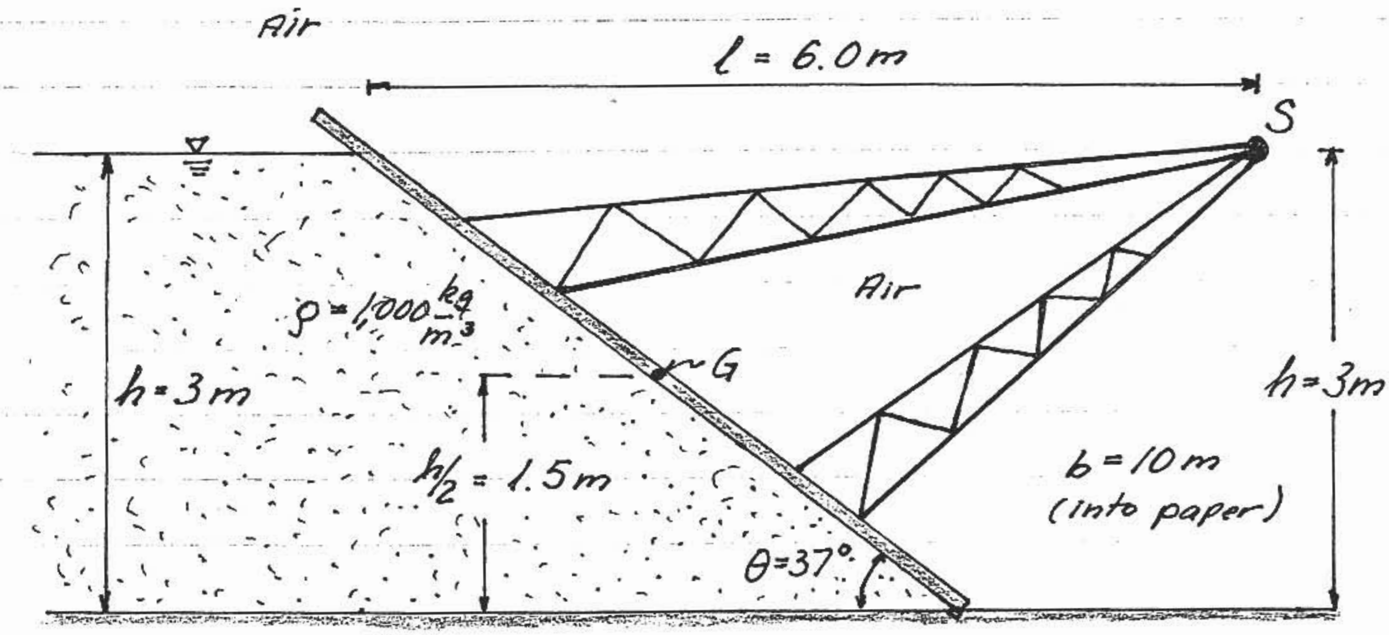
a) Determine the average velocity in the 10-cm-diameter pipe, V_p.
b) Determine the elevation of the free surface in the container, h_c, corresponding to the given problem specifications.

1.060 FLUID MECHANICS

In-class Examination No: 1, Spring 2001

SOLUTIONS

Problem No: 1



a) Total horizontal pressure force on gate = Horizontal force on gate's projection on vertical plane = $\frac{1}{2}\rho g h^2 b = \frac{1}{2} 1{,}000 \cdot 9.8 \cdot 3^2 \cdot 10 = \underline{441\ kN} = \underline{F_H}$

Acting <u>towards the right</u> (on gate), $\underline{a_H} = h/3 = 1.0\,m$ above bottom

b) Total vertical pressure force on gate = weight of fluid above gate's translation into channel =
$\frac{1}{2}\rho g h (h \cot\theta) b = \frac{1}{2}\rho g h (1.33h) b = \frac{1}{2} 1000 \cdot 9.8 \cdot 3 \cdot 4 \cdot 10 = \underline{588\ kN} = \underline{F_V}$

Acting <u>upwards</u> (on gate), $\underline{a_V} = \frac{2}{3} h \cot\theta = \underline{2.67\,m}$ to the <u>right</u> of gate's intersection with still water level.

c) Taking moment of pressure forces and gate's weight around shaft at S

$M_V = F_V\, a_{VS} = F_V\,(l - a_V) = 588\,(6-2.67) = 1{,}958$ kNm CW

$M_H = F_H\, a_{HS} = F_H\,(h - a_H) = 441\,(3-1) = 882$ kNm CCW

$M_{W_g} = W_g\,(l - \frac{1}{2} h \cot\theta) = W_g\,(6 - \frac{1}{2} 3 \cdot 1.33) = 4\,W_g$ kNm CCW

Balance of Moments gives required weight [in kN]

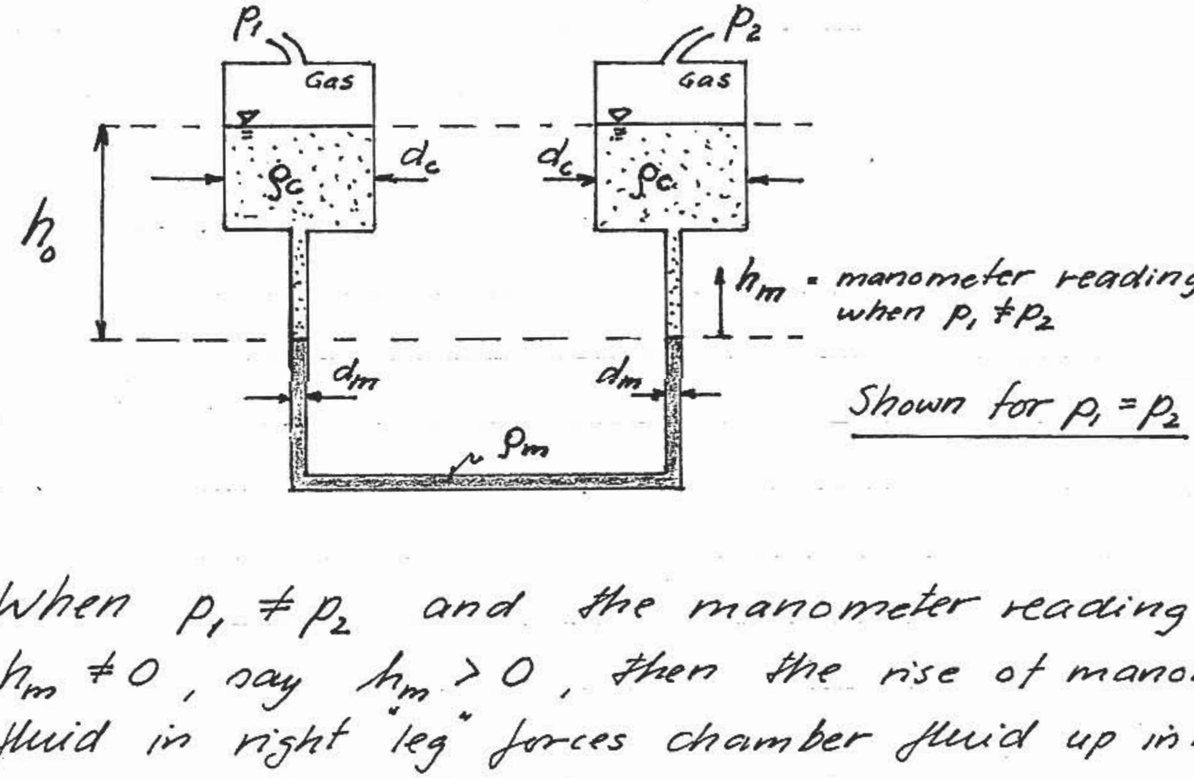
$M_V = M_H + M_{W_g} \Rightarrow \underline{W_g} = \frac{1}{4}(M_V - M_H) = \frac{1}{4} 1{,}076 = \underline{269 \text{ kN}}$

d) Horizontal force on S = F_H = 441 kN (to the right)

Vertical force on S = $F_V - W_g$ = 588 − 269 = 319 kN (upwards)

<u>Total force on shaft</u> = $\sqrt{441^2 + 319^2}$ = <u>544 kN</u>

<u>Problem No: 2</u>



When $p_1 \neq p_2$ and the manometer reading $h_m \neq 0$, say $h_m > 0$, then the rise of manometer fluid in right "leg" forces chamber fluid up into right chamber. Also, if manometer fluid rises in right leg by h_m, it drops in left leg by h_m and the level in left chamber drops

a) Assuming h_m given we have the corresponding change h_c in the chamber from conservation of volume of chamber fluid

$$h_m \frac{\pi}{4} d_m^2 = h_c \frac{\pi}{4} d_c^2 \Rightarrow h_c = h_m (d_m/d_c)^2$$

Now starting at original equilibrium level in chamber 1 using manometry rules

$$p_1 + \rho_c g (h_m - h_c + h_0) - \rho_m g\, 2h_m - \rho_c g (h_0 - h_m + h_c) = p_2$$

or

$$\underline{p_1 - p_2} = 2(\rho_m - \rho_c) g h_m + 2\rho_c g h_c = \underline{2(\rho_m - \rho_c) g h_m + 2\rho_c g \left(\frac{d_m}{d_c}\right)^2 h_m}$$

Notice: If $\rho_m \approx \rho_c$ the diameter ratio d_m/d_c must be very small to allow neglect of second term! This is clearly seen by writing the above formula

$$\underline{(p_1 - p_2) = 2\rho_c g h_m \left(\frac{\rho_m}{\rho_c} - 1 + \left(\frac{d_m}{d_c}\right)^2\right)}$$

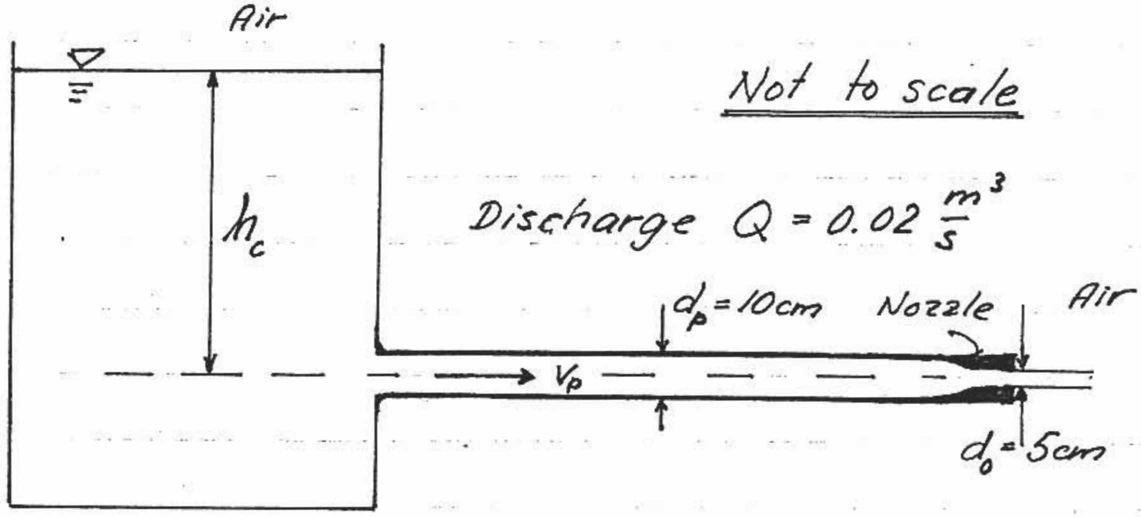
b) For values given we have

$$\underline{p_1 - p_2} = 2 \cdot 970 \cdot 9.8 \cdot 0.05 \left(\frac{1000}{970} - 1 + \left(\frac{1}{10}\right)^2\right) =$$

$$950.6\,(0.0309 + 0.01) = \underline{38.9\ \text{Pa}}$$

[Note: This pressure difference corresponds to only 4 mm of water column equivalent]

Problem No: 3



a) $Q = Q_p = A_p V_p \ ; \quad A_p = \frac{\pi}{4} d_p^{\,2} = 7.85 \cdot 10^{-3}\ m^2$

$$\underline{V_p} = \frac{Q_p}{A_p} = \frac{0.02}{7.85 \cdot 10^{-3}} = \underline{2.55\ m/s}$$

b) Velocity at nozzle exit $= V_0 = V_p \left(d_p / d_0\right)^2 =$
$V_p\ (10/5)^2 = 4\ V_p = 10.2\ m/s$
Pressure in jet at nozzle exit $= p_{atm} = 0 = p_0$
Elevation at nozzle exit $= z_0 = 0$ (by definition)
Bernoulli from free surface in very large container to nozzle exit gives

$$\frac{1}{2}\rho V_0^2 + 0 + 0 = \frac{1}{2}\rho \underbrace{V_c^2}_{\substack{0 \\ \text{(large container)}}} + \overbrace{p_c}^{p_{atm} = 0} + \rho g z_c = \rho g h_c$$

$$\underline{h_c} = \frac{V_0^2}{2g} = \frac{(10.2)^2}{2 \cdot 9.8} = \underline{5.30\ m}$$