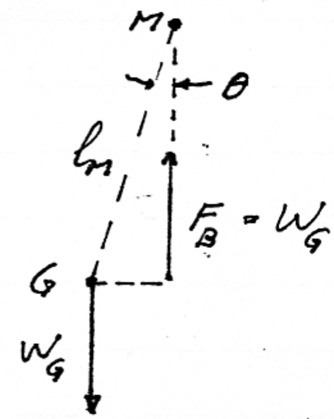
RECITATION # 7 –

DYNAMIC RESPONSE OF A FLOATING BODY

(continuation of Recitation #2 – Stability of a Floating Body)

In assessing the conditions for stability of a floating body we considered the static situation, i.e. we rotated the body a small angle around its center of gravity and, keeping it fixed in the rotated position, we obtained the forces acting on the body to be a pure moment shown below.



With

$$MG = z_M' - z_G = l_M \tag{17}$$

and denoting the small angular rotation by θ (replacing $\delta\theta$) the restoring moment around G was found to be

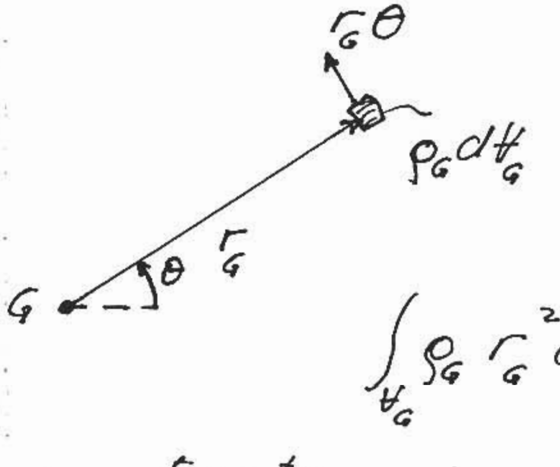
$$\overset{\curvearrowleft}{M}_{RG} = F_B\,(z_M' - z_G)\sin\theta \simeq$$

$$M_G\, g\, l_M\, \theta \tag{18}$$

To obtain the body's response to this restoring moment, we apply conservation of angular momentum for the solid body's rotation around its center of gravity, G.

$$\left(\int_{\forall_G} \rho_G r_G^2 d\forall_G\right)\ddot{\theta} + M_G g l_M \theta = 0 \qquad (19)$$

where $\forall_G$ is the volume of the material making up the body, and r_G is the distance of any point of the body from the center of gravity, and $(\dot{\ })$ denotes differentiation with respect to time, i.e. $(\dot{\ }) = d()/dt$.



Formally, we may write the moment of inertia in the form

$$\int_{\forall_G} \rho_G r_G^2 d\forall_G = M_G R_G^2 \qquad (20)$$

i.e. treating is as a lumped mass, M_G, located a distance R_G = the inertial radius from the center of gravity

With (19) and (20) the equation governing the response of the body to a restoring moment becomes

$$M_G R_G^2 \ddot{\theta} + M_G g l_M \theta = 0$$

or

$$\ddot{\theta} + \frac{l_M}{R_G^2} g \theta = 0$$

or

$$\ddot{\theta} + \omega^2 \theta = 0 \qquad (21)$$

with

$$\omega^2 = (l_M / R_G^2) g \qquad (22)$$

The general solution to the ODE, (21), is

$$\theta = \theta_c \cos\omega t + \theta_s \sin\omega t \qquad (23)$$

where θ_c and θ_s are arbitrary constants to be determined from the initial conditions.

With initial conditions corresponding to the release of the body in its rotated position these are

$$\theta = \theta_m \; ; \; \dot{\theta} = 0 \text{ , at } t = 0 \qquad (24)$$

and we have

$$\theta = \theta_m \cos \omega t \tag{25}$$

Thus, the dynamic response of a free floating body that has been displaced from its vertical equilibrium by an angular rotation is a simple oscillation of period

$$T = 2\pi/\omega = 2\pi \frac{R_G}{\sqrt{g l_M}} \tag{26}$$

The period of oscillation is a measure of how fast the body will return to its original vertical position. From (26) we see that the response time decreases as R_G decreases (smaller R_G corresponds to a smaller inertia moment and hence less "resistance" or "sluggishness" in body's response) and as l_M increases (larger l_M increases the magnitude of the moment M_R acting to restore the body to its original vertical position).

Since our analysis of the dynamic response of the floating body is HIGHLY SIMPLIFIED (neglect of dynamic effects from the fluid surrounding the submerged part of the body) the period T = natural period of the body's rotation is best determined from experiments.

The preceding analysis of the dynamic response of a floating body did not consider any externally applied force (moment). Let us now consider a floating body that experiences an externally applied "force" in the form of an oscillating moment (could, for example come from waves on the surface of the surrounding fluid hitting the floating body). Corresponding to this excitation the equation governing the body's response is formally

$$\ddot{\theta} + \omega^2 \theta = m_e \cos \omega_e t \qquad (27)$$

where m_e represents the external forcing moment.

The particular solution to (27) is simply

$$\theta_p = \theta_e \cos \omega_e t \qquad (28)$$

with θ_e obtained from (27) as

$$\theta_e = \frac{m_e}{\omega^2 - \omega_e^2} \qquad (29)$$

Thus, the floating body's response

to an externally applied oscillatory forcing is simply an oscillatory motion of the same period as the forcing and an amplitude scaled by the magnitude of the external forcing (m_e). However, we also see from (29) that the magnitude of the response, θ_e, depends on the difference between the period of the forcing, $T_e = 2\pi/\omega_e$, and the natural period of the floating body, $T = 2\pi/\omega$. In fact, the response becomes infinite, regardless of m_e so long as it is not 0, when $T_e = T$ (or $\omega = \omega_e$). Of course, $\theta_e \to \infty$ violates the assumption that $\theta \ll 1$, so our theory is no longer valid. Nevertheless, the prediction of "infinite" response when $\omega_e \to \omega$ signifies that trouble can be antici-pated.

This behavior of a dynamic response to an oscillating forcing is referred to as <u>RESONANCE</u> ($\omega_e \to \omega$) and must be avoid even if the static analysis has shown that the floating body is stable against rotations.

SUMMARY OF FLOATING BODY STABILITY

1) Metacentric height

$$z_M = \frac{I_{yy}}{\forall_B} + z_B$$

must be larger than height of center of gravity of floating body, z_G, for stability.

2) The larger

$$l_M = z_M - z_G$$

the more stable the body is - and the faster the body will return to its vertical equilibrium position.

3) Subjected to an oscillatory external forcing of period T_e, the response of the floating body may become dangerously large if

$$T_e \rightarrow T \quad \text{(Resonance)}$$

where T = natural period of oscillation of the floating body.