1.060 Engineering Mechanics II Spring 2006

Recitation 8 - Problems

April 20th and 21st

Problem 1

Water flows in a long straight channel whose trapezoidal cross section shown in Figure 1. The horizontal bottom is finished concrete and the sides are weedy. The bottom slope is $S_0 = 0.001$. The water depth over the horizontal bottom is 1 m.

a) Determine the discharge, Q.

b) Determine the corresponding mean velocity, V.

c) Determine the Froude number, Fr, of the flow.

d) Is the flow super- or subcritical?

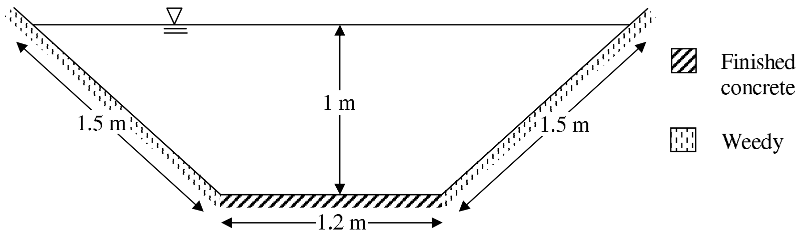


Figure 1: Trapezoidal channel in Problem 1.

Problem 2

A long glass-walled laboratory flume has a rectangular cross-section of width 1 m, a slope of $S_0 = 10^{-4}$, and it discharges water at a flowrate $Q = 0.5\ m^3/s$.

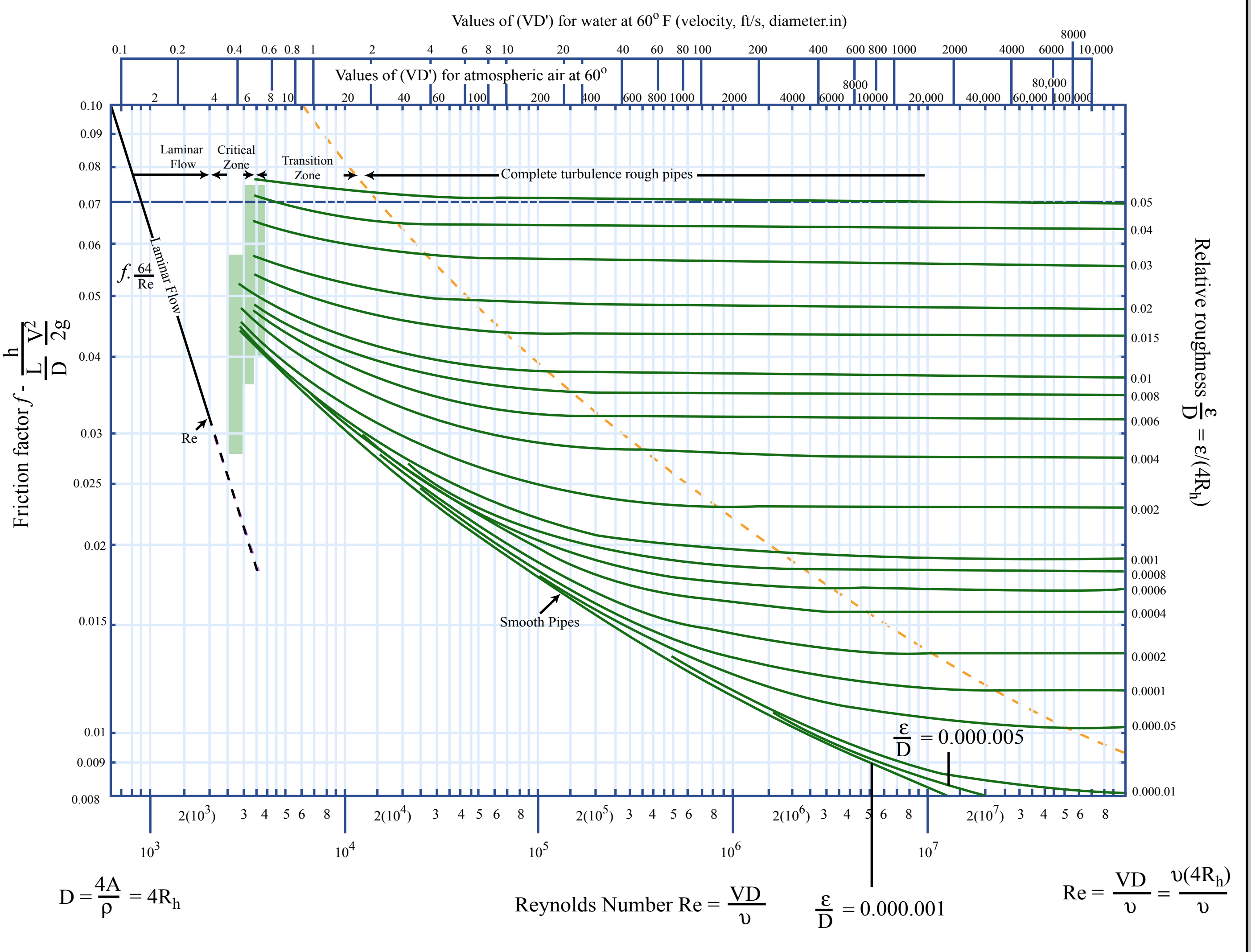
a) Find the critical depth, h_c.

b) Find the normal depth, h_n, corresponding to steady uniform flow.

c) Does the normal depth correspond to super- or subcritical flow?

Values of the Manning Coefficient, *n* (Ref 5)	
Wetted Perimeter	***n***
Natural Channels	
Clean and straight	*0.030*
Sluggish with deep pools	*0.040*
Major rivers	*0.035*
Floodplains	
Pasture, Farmland	*0.035*
Light brush	*0.050*
Heavy brush	*0.075*
Trees	*0.15*
Excavated Earth Channels	
Clean	*0.022*
Gravelly	*0.025*
Weedy	*0.030*
Stony, Cobbles	*0.035*
Artifically lined channels	
Glass	*0.010*
Brass	*0.011*
Steel, Smooth	*0.012*
Steel, Painted	*0.014*
Steel, Riveted	*0.015*
Cast iron	*0.013*
Concrete, Finished	*0.012*
Concrete, Unfinished	*0.014*
Planed Wood	*0.012*
Clay tile	*0.014*
Brickwork	*0.015*
Asphalt	*0.016*
Corrugated metal	*0.022*
Rubble masonry	*0.025*

Table by MIT OCW.

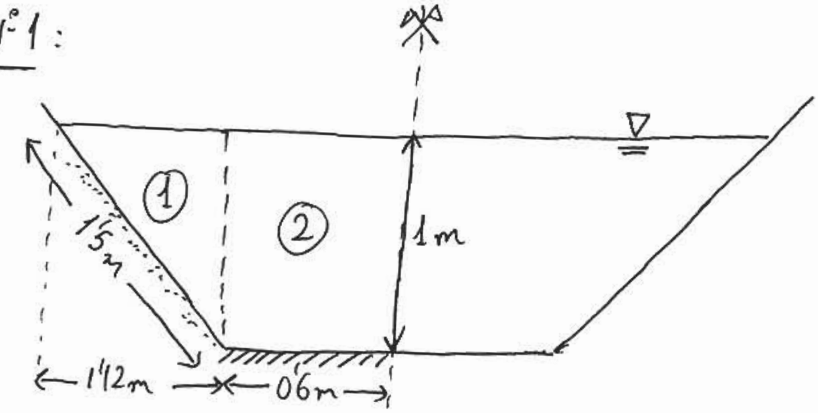


The Moody chart for pipe friction with smooth and rough walls. This chart is identical to Eq. (6.64) for turbulent flow.

Graph by MIT OCW.

RECITATION 8 - SOLUTIONS

- PROBLEM N° 1:



a) Since the channel has a symmetric section, we can study one half of it:

Areas: $A_1 = \frac{1}{2} \cdot 1 \cdot 1'12 = 0'559\ m^2$, $A_2 = 0'6 \cdot 1 = 0'6\ m^2$

Wetted perimeters: $P_1 = 1'5\ m$, $P_2 = 0'6\ m$

Hydraulic radii: $R_{H_1} = A_1/P_1 = 0'373\ m$, $R_{H_2} = A_2/P_2 = 1\ m$

Manning's "n": $n_1 = 0'030$, $n_2 = 0'012$ (From Young et al., Table 10.1)

Manning: $Q_1 = A_1 \frac{1}{n_1} (R_{H_1})^{2/3} \sqrt{S_0} = 0'559 \frac{1}{0'030} (0'373)^{2/3} \sqrt{10^{-3}} = 0'305\ m^3/s$

$Q_2 = A_2 \frac{1}{n_2} (R_{H_2})^{2/3} \sqrt{S_0} = 0'6 \frac{1}{0'012} 1^{2/3} \sqrt{10^{-3}} = 1'581\ m^3/s$

$Q = 2(Q_1 + Q_2) = 3'77\ m^3/s$

b) Total area: $A = 2(A_1 + A_2) = 2'32\ m^2$, Average velocity: $V = \frac{Q}{A} = \frac{3'77}{2'32} = 1'63\ m/s$

c) Mean depth: $h_m = \frac{A}{b_s} = \frac{2'32}{1'2 + 2 \cdot 1'12} = 0'674\ m$; $Fr = \frac{V}{\sqrt{g h_m}} = \frac{1'63}{\sqrt{9'8 \cdot 0'674}} = 0'63$

d) $Fr = 0'63 < 1 \Rightarrow$ SUBCRITICAL FLOW

- PROBLEM N° 2:

a) $Fr = \frac{Q/(b \cdot h_c)}{\sqrt{g h_c}} = 1 \Rightarrow h_c = \left(\frac{(Q/b)^2}{g} \right)^{1/3} = \left(\frac{(0'5/1)^2}{9'8} \right)^{1/3} = 0'29\ m$

b) Since the flume has glass walls, $\varepsilon \simeq 0 \Rightarrow$ Smooth turbulent flow, and we should use Darcy-Weisbach (and not Manning).

Darcy-Weisbach: $Q = A \cdot V = A\sqrt{\frac{8g}{f} R_h S_0} \Rightarrow 0'5 = h\sqrt{\frac{8 \cdot 9'8}{f}}\sqrt{\frac{h}{1+2h} 10^{-4}} \Rightarrow$

$\Rightarrow \frac{h^3}{1+2h} = 31'9 f$. Solve by iteration:

<u>1ST ITERATION</u>: $f_0 = 0'02 \Rightarrow \frac{h_1^3}{1+2h_1} = 0'638 \Rightarrow h_1^{(k+1)} = \left[0'638\left(1+2h_1^{(k)}\right)\right]^{1/3}$

Solve for h_1 by iteration, with $h_1^{(0)} = 0$

k	0	1	2	3	4	5	6
$h_1^{(k)}$	0	0'861	1'202	1'295	1'318	1'324	1'325

$\Rightarrow h_1 = 1'33 \, m$

$Rh_1 = \frac{1'33}{1+2 \cdot 1'33} = 0'363 \, m$, $V_1 = \frac{0'5}{1 \cdot 1'33} = 0'376 \, m/s$

$Re_1 = \frac{V_1 (4Rh_1)}{\nu} = \frac{0'376 \cdot (4 \cdot 0'363)}{10^{-6}} = 5'46 \cdot 10^5 \overset{MOODY}{\Longrightarrow} f_1 = 0'0128$

<u>2ND ITERATION</u>: $f_1 = 0'0128 \Rightarrow \frac{h_2^3}{1+2h_2} = 0'408 \Rightarrow h_2^{(k+1)} = \left[0'408\left(1+2h_2^{(k)}\right)\right]^{1/3}$

Solve for h_2 by iteration, with $h_2^{(0)} = h_1 = 1'33 m \Rightarrow h_2 = 1'09 \, m$

$Rh_2 = 0'343 \, m$, $V_2 = 0'459 \, m/s$, $Re_2 = 6'30 \cdot 10^5 \overset{MOODY}{\Longrightarrow} f_2 = 0'0125$

<u>3RD ITERATION</u>: $f_2 = 0'0125 \Rightarrow \frac{h_3^3}{1+2h_3} = 0'399 \Rightarrow h_3^{(k+1)} = \left[0'399\left(1+2h_3^{(k)}\right)\right]^{1/3}$

Solve for h_3 by iteration, with $h_3^{(0)} = h_2 = 1'09 m \Rightarrow h_3 = 1'08 \, m$ DONE

The normal depth is $\underline{\underline{h_n = 1'08 \, m}}$

If we had applied Manning (which is unjustified, since flow is not rough turbulent) with $n = 0'010$ (from Table 10.1 in Young et al.):

$Q = A \frac{1}{n} (Rh)^{2/3} \sqrt{S_0} \Rightarrow 0'5 = h \frac{1}{0'010} \left(\frac{h}{1+2h}\right)^{2/3} \sqrt{10^{-4}} \Rightarrow h = 0'660 (1+2h)^{2/5}$

Solve for h by iteration, with $h^{(0)} = 0$ and $h^{(k+1)} = 0'660\left(1+2h^{(k)}\right)^{2/5}$, to obtain $h_n = 1'03 \, m$ → Pretty good approximation in this case.

c)

$h_n = 1'08 \, m > h_c = 0'29 \, m \Rightarrow$ <u>SUBCRITICAL FLOW</u>