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1.061 / 1.61 Transport Processes in the Environment
Fall 2008

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Answer 6.1

Station 1: A 50-mg/l solution of tracer is injected at the rate of $Q_i = 100 \text{ cm}^3/\text{s}$.

Station 2: Located 100-m downstream of Station 1.
Dye concentration, $C_{\text{dye}} = 10 \text{ } \mu\text{g/l}$
Lindane concentration, $C_{L2} = 0.5 \text{ } \mu\text{g/l}$

Station 3: Located 200-m downstream of Station 1.
Dye concentration, $C_{\text{dye}} = 8 \text{ } \mu\text{g/l}$
Lindane concentration, $C_{L3} = 0.9 \text{ } \mu\text{g/l}$

If dye is well-mixed and concentration is steady, then the flow at stations 2 and 3 is

$$Q_2 = Q_i C_i / C_{\text{dye}} = (100 \text{ cm}^3 \text{ s}^{-1})(50 \text{ mg l}^{-1}) / (0.01 \text{ mg l}^{-1}) = 0.50 \text{ m}^3 \text{ s}^{-1}$$

$$Q_3 = Q_i C_i / C_{\text{dye}} = (100 \text{ cm}^3 \text{ s}^{-1})(50 \text{ mg l}^{-1}) / (0.008 \text{ mg l}^{-1}) = 0.63 \text{ m}^3 \text{ s}^{-1}$$

If no tributaries or sewer outfalls exist between stations 2 and 3, then we expect the groundwater inflow between these stations to be

$$Q_{\text{GW}} = Q_3 - Q_2 = 0.13 \text{ m}^3 \text{ s}^{-1}.$$

To find the Lindane concentration in the groundwater we evaluate conservation of mass within the river between station 2 and 3. Influxes of Lindane to this control volume are at station 2 and from groundwater. The outflow is at station 3. Such that

$$Q_2 C_{L2} + Q_{\text{GW}} C_{\text{LGW}} = Q_3 C_{L3}$$

Solving for the groundwater concentration

$$C_{\text{LGW}} = \frac{Q_3 C_{L3} - Q_2 C_{L2}}{Q_{\text{GW}}} = 2.4 \text{ } \mu\text{g/l}$$

Answer 6.2

Case 1. The safety doors are closed so that there is no air current in the hall.

Use time scales to determine if, for the exposure at your office, the release is instantaneous or continuous. The distance to your door is $dx=10\text{-m}$. With no advection (zero current), the transport time scale is $T_{\text{diffusion}}=dx^2/8D$, based on the arrival of the 2σ contour. Thus, $T_{\text{diffusion}}=(10\text{-m})^2/(8 \times 0.05\text{m}^2\text{s}^{-1})=250\text{ s}$. This is much longer than the release time scale (25 s), so the concentration observed at your office will appear as if from an instantaneous release.

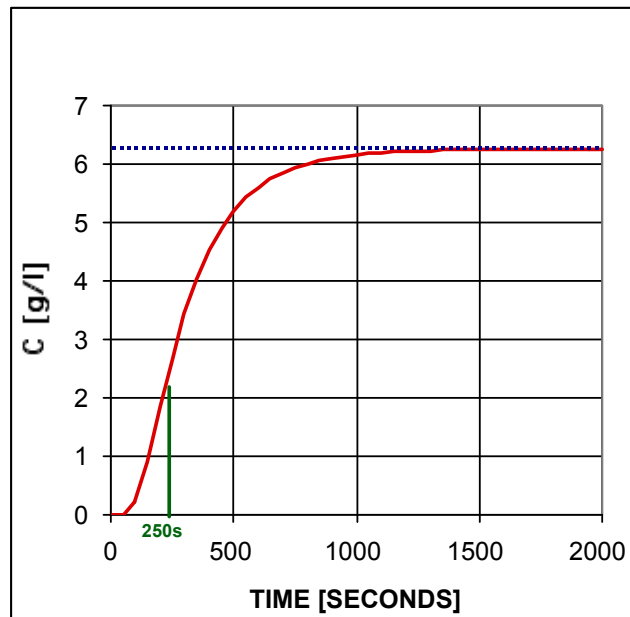
Similarly, use a comparison of time-scales to determine if the concentration at your door appears as if from a 1-D, 2-D, or 3-D source. If we assume the release occurs at mid-width, the time-scale to mix across the hallway is, $t_{\text{mix},y}=(2\text{m})^2/(4 \times 0.05 \text{ m}^2\text{s}^{-1})=20\text{ s}$. For the vertical mixing time-scale, note that the spill is at the floor boundary. A release at the boundary must diffuse across twice the distance, as compared to a release at mid-height, so the length-scale must be doubled. The time-scale to mix over the hall height is thus, $t_{\text{mix},z}=(2 \times 2\text{m})^2/(4 \times 0.05\text{m}^2\text{s}^{-1})=80\text{s}$. Since $t_{\text{diffusion}} \gg t_{\text{mix}}$, the concentration at your door appears as if released as a one-dimensional source, *i.e.* distributed evenly over y and z .

Finally, with the closed doors at $x = \pm 20$ are no-flux boundaries. To account for these boundaries we put in an infinite set of images located at $x = 40n$, where $n = \pm 1, \pm 2$, etc. The real source is located at $n=0$. The concentration observed at your door then follows,

$$C(x,t) = \frac{M}{A\sqrt{4\pi Dt}} \sum_{n=-\infty}^{\infty} \left(\exp\left(-\frac{(x+40n)^2}{4Dt}\right) \right),$$

with $x = 10\text{ m}$. The cross-sectional area of the hallway is $A = 4\text{ m}^2$.

With the doors closed, the hallway is a contained system, and the final concentration is simply that established when the mass released is fully mixed over the volume of the hallway. We assume that the doors to individual rooms are closed, so that no fumes enter the rooms. The final concentration is then, $C_{\text{final}} = 1\text{ g} / (2 \times 2 \times 40\text{ m}^3) = 6.25\text{ g/l}$.



The concentration observed at your door ($x = 10$ m) is plotted above. Note the concentration at your door at the diffusion time scale, 250 s, is not negligible. This time scale is based on the arrival of the 2σ contour, not on a specific value of concentration. The time-scale based on cloud size (2σ) is not necessarily a good indicator of exposure risk. Also, if the absolute concentration at 2σ is below the detection limit, you would not detect it, even though you predict it has arrived. The final concentration is reached at around 1600 s, which is shorter than $t_{\text{mix},x} = L_x^2/4D = 8000$ s, with $L_x = 40$ m (defined in chap. 4). This again emphasizes the conservative nature of the standard mixing-time estimator, $L^2/4D$.

Case 2. Safety doors are open, $u = 4$ m/s from the spill towards your door.

The advection time scale is, $T_{\text{advection}} = dx/u = 10\text{-m}/4\text{ms}^{-1} = 2.5$ s $\ll T_{\text{diffusion}}$ and $\ll T_{\text{release}}$. So, the transport is dominated by advection, and the chemical arrives at your door before the release ends. So, during most of the release the concentration at your door appears as if released from a continuous source. In addition, the mixing time scales derived above still hold, so we can write, $T_{\text{mix},y}$ and $T_{\text{mix},z} > T_{\text{advection}}$, such that the chemical cloud is not mixed over y and z at the door. The cloud is thus fully three-dimensional at your door. For simplicity we assign the origin of the release to be ($x = 0, y = 0, z = 0$). Image sources are needed to account for the no-flux boundaries at $y = \pm 1$ -m and $z = 0$ and 2 m. Consider the four closest images, $(y,z) = [(0,0); (-2,0); (2,0); (0,4)]$. During most of the release, the concentration at your door will be,

$$C(x, y, z) = \frac{\dot{m}}{4\pi D x} \left[\underset{\text{real + image at } (y=0,z=0)}{2 \exp\left(-\frac{u(y^2 + z^2)}{4Dx}\right)} + \underset{\text{image at } (y = 2,z = 0)}{\exp\left(-\frac{u((y - 2)^2 + z^2)}{4Dx}\right)} + \underset{\text{image at } (y = -2, z = 0)}{\exp\left(-\frac{u((y + 2)^2 + z^2)}{4Dx}\right)} + \underset{\text{image at } (y = 0, z = 4)}{\exp\left(-\frac{u(y^2 + (z - 4)^2)}{4Dx}\right)} \right]$$

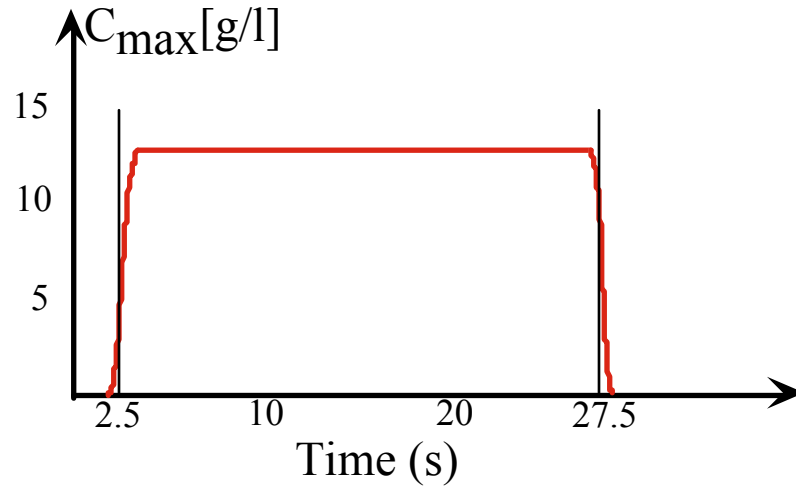
The maximum concentration at the door will be at the floor - in line with the source. So, we evaluate the above equation for ($x = 10$ m, $y = 0, z = 0$).

$$C_{\text{max}} = \frac{\dot{m}}{4\pi D x} \left[2 + 2 \exp\left(-\frac{u(2\text{m})^2}{4Dx}\right) + \exp\left(-\frac{u(4\text{m})^2}{4Dx}\right) \right] =$$

$$= \frac{(1\text{g}/25\text{s})}{4\pi(0.05\text{m}^2\text{s}^{-1})(10\text{m})} \left[2 + 2(0.00034) + 1.2 \times 10^{-14} \right] = 12.7\text{g/l}$$

One can see from the values in the last expression that only the image coincident with the real source contributes significantly to the solution. The above concentration is observed at the door only during the duration of the release. More specifically, the concentration is observed starting approximately at $t = 2.5$ s (the advection time scale for the front) and ending at $(25 + 2.5) = 27.5$ seconds, the duration of the release plus the advection time scale. The maximum exposure concentration at your door is depicted below. Because of longitudinal diffusion, the front and tail of the passing cloud are not sharp, but will have a transition region of length $\approx 4\sigma$, where σ is estimated at the advection time-scale, 2.5s. Thus, the transition region at the front and tail of the exposure cloud will $4 \sqrt{2(0.05\text{m}^2\text{s}^{-1})(2.5\text{s})} = 2$ m long, which translates into an transient

period of $(2\text{-m})/(4\text{m/s}) = 0.5\text{ s}$. Note that the final concentration is zero, everything eventually flushes out of the hall by the air current.

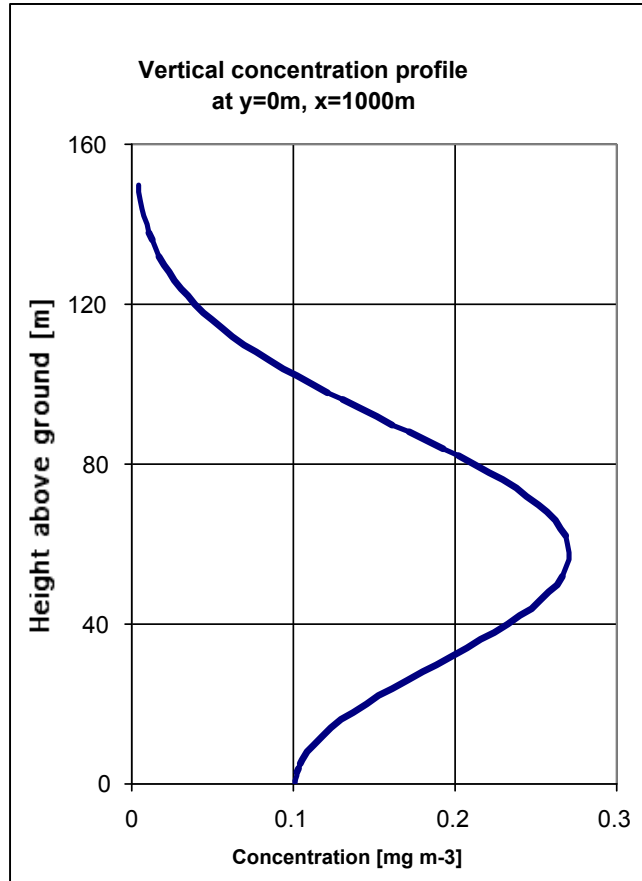


Answer 6.3.

The stack will run continuously, so use a continuous release solution. Let $H = 57.9\text{-m}$ be the stack height. With $z = 0$ at the ground, the source is at $(x=0, y=0, z = H)$. One image source is needed at $(x = 0, y = 0, z = -H)$ to account for the no-flux boundary at the ground.

$$C(x, y, z) = \frac{\dot{m}}{4\pi x \sqrt{D_y D_z}} \left[\underbrace{\exp\left(-\frac{uy^2}{4D_y x} - \frac{u(z-H)^2}{4D_z x}\right)}_{\text{real source}} + \underbrace{\exp\left(-\frac{uy^2}{4D_y x} - \frac{u(z+H)^2}{4D_z x}\right)}_{\text{image source}} \right]$$

This equation is valid when $Pe=Ux/D_x \gg 1$. This is true for $x \gg D_x/U = 1\text{-m}$. Since the house is directly downwind, we let $y = 0, x = 1000\text{-m}$ in the above equation. The rate of release is, $\dot{m} = 150,000\text{kg/yr} = 4.8\text{ gs}^{-1}$. See plot of $C(z)$ below. The maximum concentration at your home will be at the roof, $C(z = 10\text{-m}) = 0.11\text{ mg/m}^3 = 0.11\text{ }\mu\text{g/l}$. Finally, note the no-flux boundary is expressed in the profile as $\partial C/\partial z = 0$ at $z = 0$.

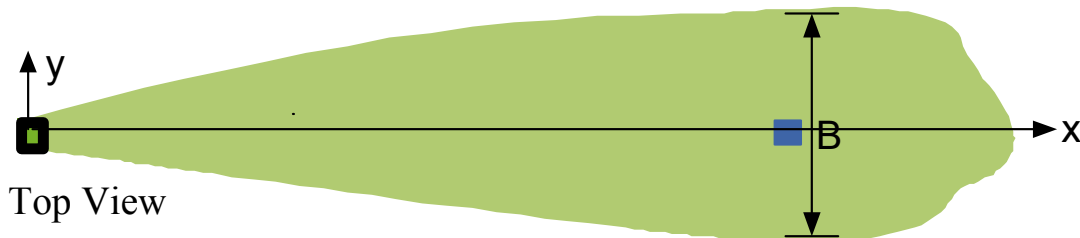


Answer 6.4

The release is described as continuous, and we are told that it rapidly distributes over the aquifer depth. No lateral boundaries are indicated. As a first approximation, we believe the concentration field will resemble that from a continuous release in two dimensions, (x,y) , with conditions uniform in the vertical coordinate, z . No image sources are needed, because boundaries only restrict flux in the z -direction, and the concentration is already assumed to be constant in z , such that the no-flux condition, $\partial C/\partial z = 0$, is automatically satisfied. However, the source is not a point, but rather is distributed over a 5-m x 5-m square. We can treat it as a point if this area is small compared to the extent of the plume at the well (see B below), as the concentration observed at the well will then appear as if it came from a point source. We estimate the lateral dimension of the plume as $B = 4 \sigma_y = 4 \sqrt{(2K_y L/U)}$, where $L = 500\text{m}$ is the distance to the well, and the lateral dispersivity is $K_y = 10^{-6} \text{m}^2 \text{s}^{-1}$. Then, $B = 126\text{m}$. Since $5\text{m} \ll 126\text{m}$, the concentration at the well appears as if it originated from a point source. Finally, we write the expression for brine concentration downstream of the leak.

$$C(x, y) = \frac{m/u}{L_z \sqrt{4\pi K_y (x/u)}} \exp\left(-\frac{uy^2}{4K_y x}\right),$$

with $L_z = 2\text{m}$ the thickness of the aquifer. This solution is valid for $Pe = Ux/K_x \gg 1$, or $x \gg 10\text{m}$, so we may apply it at the well located at $x = 500\text{m}$.

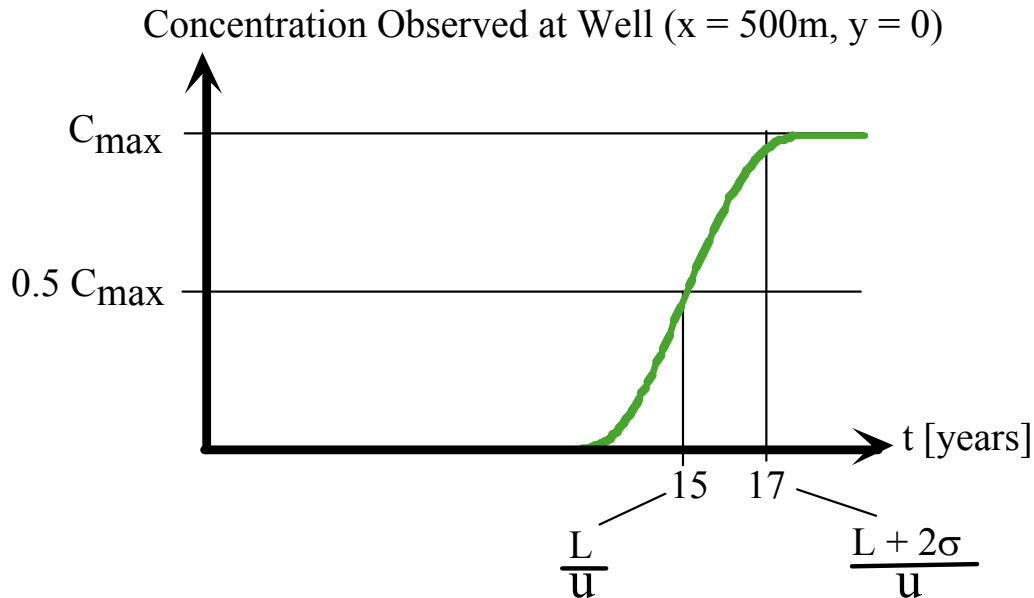


The maximum concentration at the well, C_{\max} , is given by the above equation evaluated for $(x=500\text{m}, y=0\text{m})$.

$$C_{\max} = \frac{(10\text{g/l})(0.1\text{m}^3/\text{week})(1\text{week}/604800\text{s})}{(10^{-6}\text{m/s})(2\text{m})\sqrt{4\pi(10^{-6}\text{m}^2/\text{s})(500\text{m}/10^{-6}\text{m/s})}} = 0.011\text{ g/l} = 0.011\text{ ppt.}$$

This salinity is not detectable by the human palate. For reference, salt water is 30 ppt.

The solution above assumes that steady state has been reached at the well. The time-required for this to occur depends both on advection and longitudinal dispersion. The advection time-scale, $T_U = 500\text{m}/10^{-6}\text{m/s} = 5 \times 10^8\text{ s} \approx 15\text{ years}$, defines the arrival of the mid-front which will have the concentration $C_{\max}/2$. The time interval Δt between the arrival of the mid-front and the end of the transient period is approximately $\Delta t = (2\sigma_x)/u$, where σ_x is estimated at the time the mid-front arrives at the well, $\sigma_x = \sqrt{(2K_x L/u)}$. So, $\Delta t = (2/10^{-6}\text{ms}^{-1}) \times \sqrt{(2 \times 10^{-6}\text{m}^2\text{s}^{-1} \times 500\text{m} / 10^{-6}\text{ms}^{-1})} = 6.3 \times 10^7\text{ s} = 2\text{ years}$. The concentration observed at the well is sketched below.



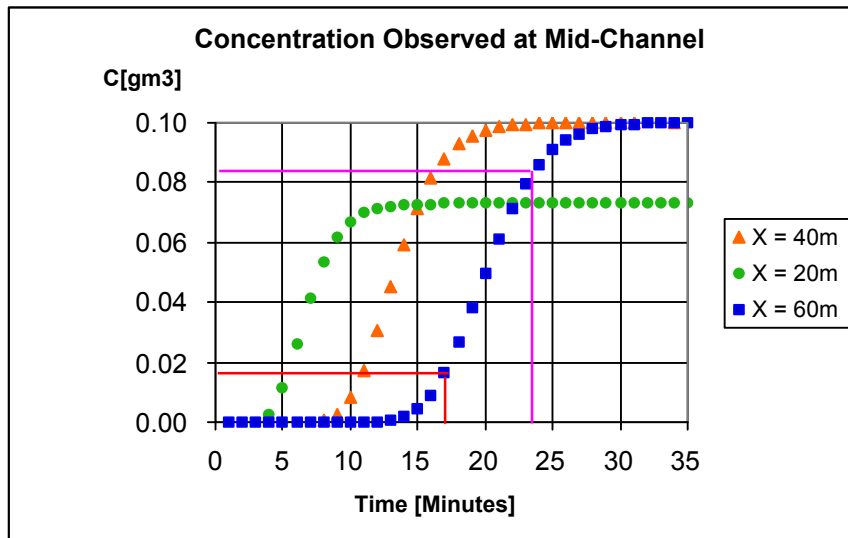
Finally, note that molecular diffusion, while present, is negligible compared to the lateral and longitudinal dispersion, and so is not considered in the plume transport calculations.

Answer 6.5

If the tracer is mixed over the channel area, $Q_r = Q_i C_i / C_f = \dot{m} / C_f$, where \dot{m} is the tracer injection rate and C_f is the steady-state, far-field concentration. From the observed concentration, and $\dot{m} = 0.01 \text{ gs}^{-1}$,

X [m]	C_f [gm ⁻³]	estimated Q_R [m ³ s ⁻¹]
20	0.075	0.13
40	0.1	0.10
60	0.1	0.10

The estimated river flow at the first station is higher than the later two stations. At first this suggests that flow is diverted from the channel between 20 and 40 m. However, diversion of flow would not change the concentration, so this is implausible. Instead, we suspect that at x = 20m the tracer has not yet fully mixed within the channel, making the observed concentration lower than the steady state value. We then conclude that the flow in the channel is $0.1 \text{ m}^3 \text{ s}^{-1}$. We estimate the Lagrangian mean velocity, u, from the arrival of the mid-front. At x = L downstream of the injection, the concentration will be $0.5C_f$ at $t = L/u$. Using the 60-m data, $C = 0.5C_f = 0.05 \text{ gm}^{-3}$ at $t = 20 \text{ minutes}$. Thus, $u = (60\text{m}) / (1200\text{s}) = 0.05 \text{ ms}^{-1}$. The mean cross-sectional area between x = 0 and x = 60-m is then, $A = Q_R / u = 2\text{m}^2$.



The coefficient of longitudinal dispersion, K_x , is estimated from the shape of the front. At $t_{0.16} = (L - \sigma_x)/u$, the concentration will be $0.16C_f$. At $t_{0.84} = (L + \sigma_x)/u$, the concentration will be $0.84C_f$. Using the concentration at $x = L = 60\text{m}$,

L [m]	u [m/s]	$t_{0.16}$ [min]	$\sigma_{0.16} = L - ut_{0.16}$ [m]	$t_{0.84}$ [min]	$\sigma_{0.84} = ut_{0.84} - L$ [m]
60	0.05	17	9	24	12

Assuming $\sigma = 0$ at $t = 0$ we have two estimates for K_x

$$K_x = \frac{1}{2} \frac{\partial \sigma^2}{\partial t} = \frac{1}{2} \left(\frac{\sigma_{0.84}^2}{t_{0.84}} \right) = \frac{1}{2} \left(\frac{(12\text{m})^2}{24\text{min}} \right) = 3.0 \text{ m}^2\text{min}^{-1}$$

$$K_x = \frac{1}{2} \frac{\partial \sigma^2}{\partial t} = \frac{1}{2} \left(\frac{\sigma_{0.16}^2}{t_{0.16}} \right) = \frac{1}{2} \left(\frac{(9\text{m})^2}{17\text{min}} \right) = 2.4 \text{ m}^2\text{min}^{-1}$$

We can average these to find the best estimate, $K_x = 2.7 \text{ m}^2\text{min}^{-1}$