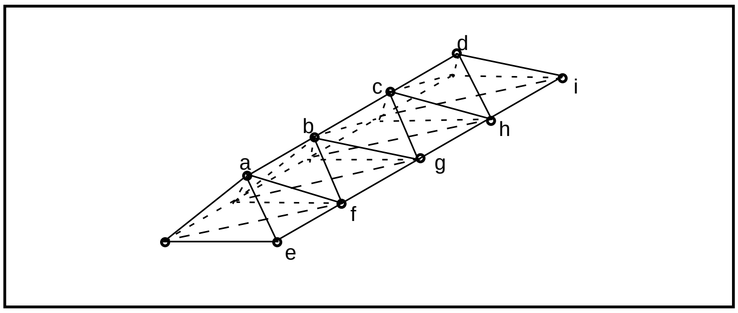
1.105 Solid Mechanics Laboratory
Fall 2003

Experiment 7   Elastic Buckling.

The objectives of this experiment are

- To study the failure of a truss structure due to local buckling of a compression member.
- To compare measurement with theoretical prediction.
- To again monitor force in one member using a strain gage and an operational amplifier and compare with theory.

The truss shown will be supported at its four corners and loaded at mid span. Two dial gages will be used to measure nodal displacements in the vertical direction at midspan (front and back). The strain gage instrumentation will measure the strain, and hence the load, in one member of the truss. The truss will be loaded until the top, midspan member “fails” due to buckling..



Experiment Procedure

Using your data from Experiment 3, check and record all relevant dimensions of the structure. *Note: The midspan, top member has been replaced with one of smaller diameter. Make sure you measure and record this dimension.*

Diameter members ab, bc, cd = __________ *in.*

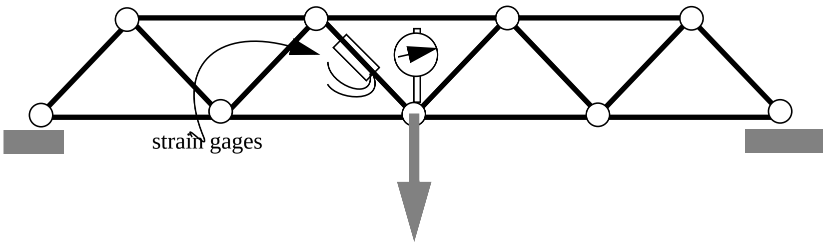
Area = ____________ *in*^{2}

Make sure the structures rests on all four support points. Adjust one of the adjustment screws at the four corners if necessary to eliminate rocking.

The bucket should be suspended by an S hook slung on the chain and centering mechanism below the mid span nodes.

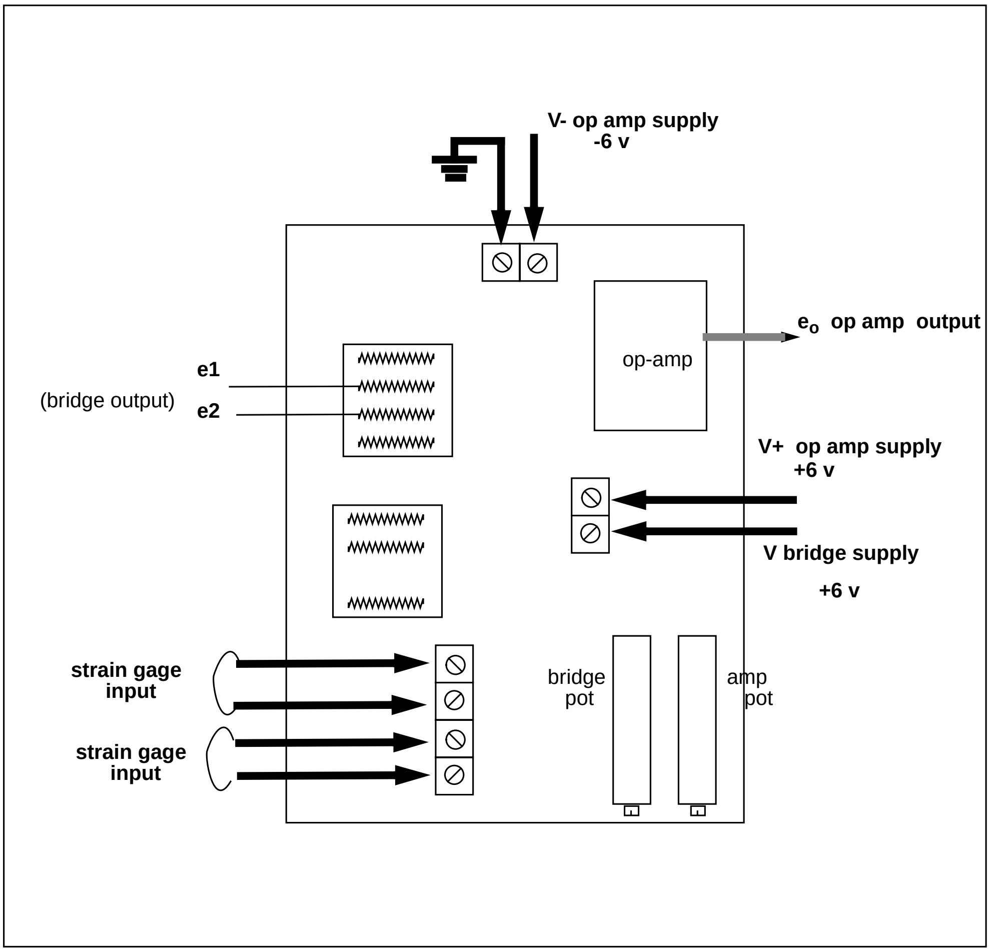
Make sure you include the stop to limit excessive vertical displacement at the mid span nodes. **Have the lab instructor check the clearance and adjust if necessary.**

You will make but one run with the truss loaded at the center nodes.



Instrumentation Preparation

Instrument preparation is much the same as it was in Experiment 3. The layout of the circuit board, showing power supply and strain gage connections is repeated here for your convenience.



Report

This document will serve as your report. Fill in the tables, plot your results on the formatted but empty graphs included, and add explanation and discussion where indicated.

Op amp Gain

e_1-e_2	e_0	Gain

Load, Deflection, OpAmp Output Data

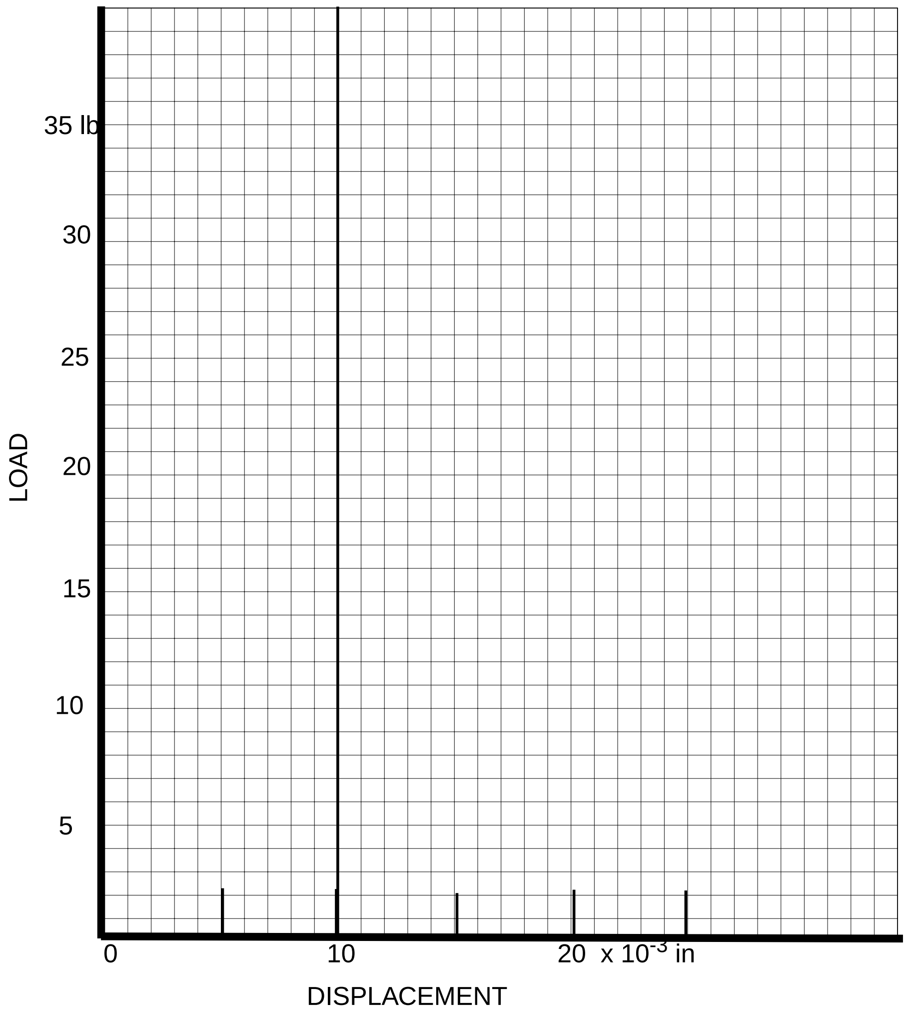
Load the structure at midspan in the increments shown. Try to keep track of the difference in displacement increments once you switch to the red weights to note any deviation from linearity.

Load		Dial gages.		Displ.	e_0	ε, Strain	Force [a]
	lbs.	x10^{-3} in		x10^{-3} in	mv		lb
	0			0	0	0	0
Bucket							

a. The area of the diagonal member is

$A_{diag.}$= __________in^2

Midspan Load Versus Midspan Displacement



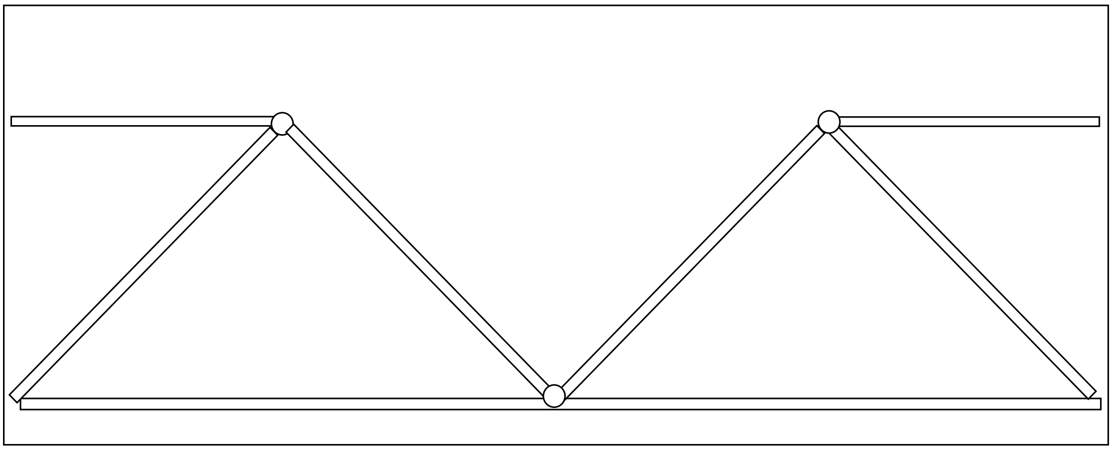
The plot above shows the measured load-displacement; it shows also the linear behavior as determined from an equivalent 2 dimensional model run on Trussworks2D. (See Appendix).

Also shown is the linear behavior of the truss as it was observed in Experiment 3, that is with the top members having the same diameter as all the others.

The values of the mid-span, applied load which would produce buckling of the top member bc, treated as a **pinned-pinned column** and as a **pinned-fixed column**, are also indicated.

Deflected Shape

Sketch the deflected shape at maximum loading conditions.

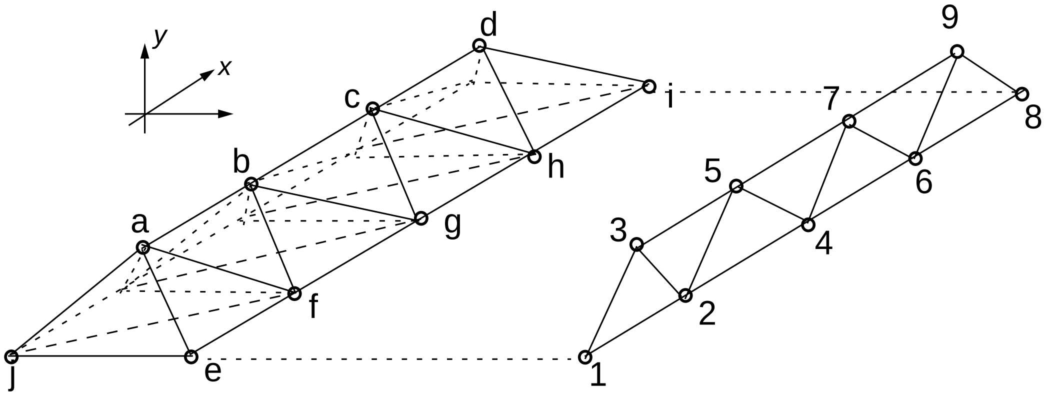


Discussion - Load/Deflection Behavior

Member Force

We compare the force in one of the diagonal members, as determined experimentally from the strain gage readings, with the predictions based upon a two-dimensional, matrix truss analysis as we attempted before.

The appendix shows how we model the three-dimensional structure as a planar truss. In this, we fix the cross sectional area of the diagonal members, 1,3; 2,3; 4,5; etc so that the stiffness of the top node, e.g., #3, in the *x* and *y* directions, is identical to that of node a in the fully three dimensional structure.



If we let A_3 be the Area of the 3D truss members, then for equivalence

$$A_2 = 1.089\ A_3$$

In the same way we attempt to replicate the stiffness of the bottom members of the 3D structure in the horizontal direction by adjusting the cross sectional area of members 1,2; 2,4; ...etc.

With these equivalent areas, and the new area of the top members as presented on page 1, a Truss2D run was conducted. From the Truss2D results, we find that the force in member 2-5 is

f_{2D} = ______________ for a midspan load of 30 lbs.

From the appendix, we have that the force in the 3 dimensional structure is related to this value by the following

f_{3D} = _________ * f_{2D}

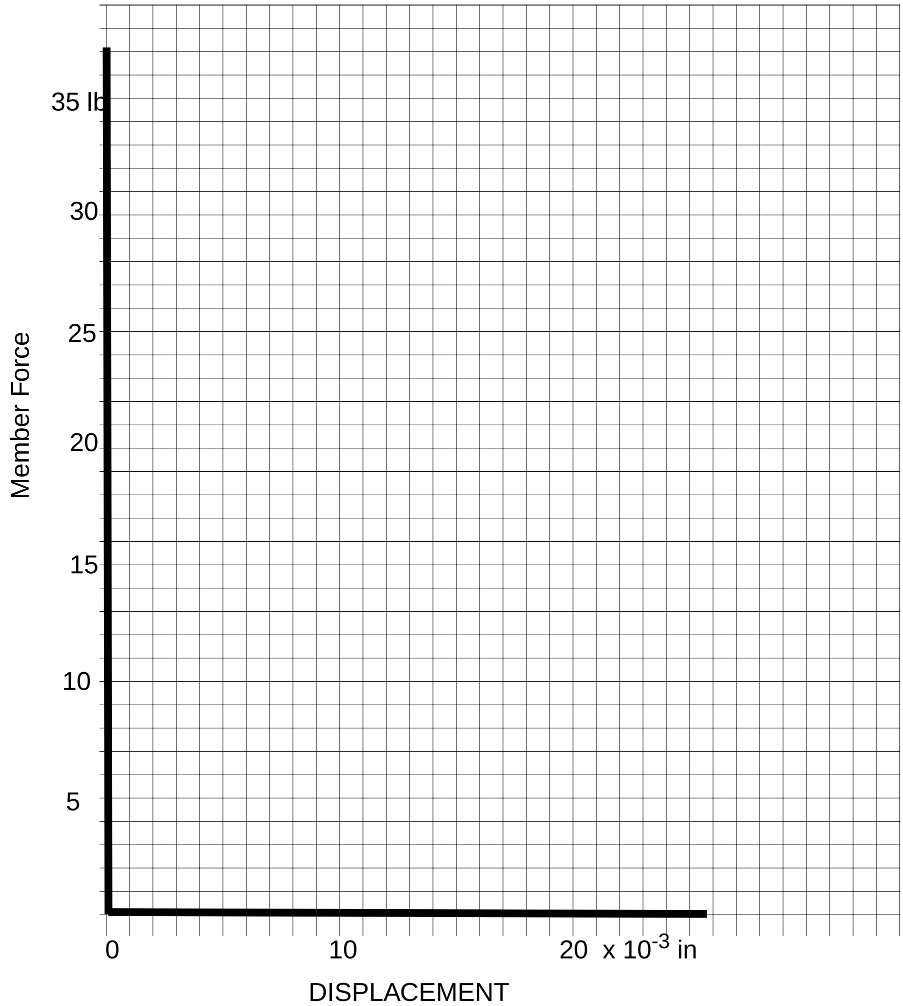
So the force in the three dimension structure, member **fb**, for a midspan load of 30 lbs is

f_{3D} = _________ lbs.

The force in member **fb** was also determined from experiment via strain gage(s) - the last column on page 3.

The plot on the next page shows how the this member force varies with displacement. The experimental data pairs are shown, so too the linear behavior as obtained from Truss2D.

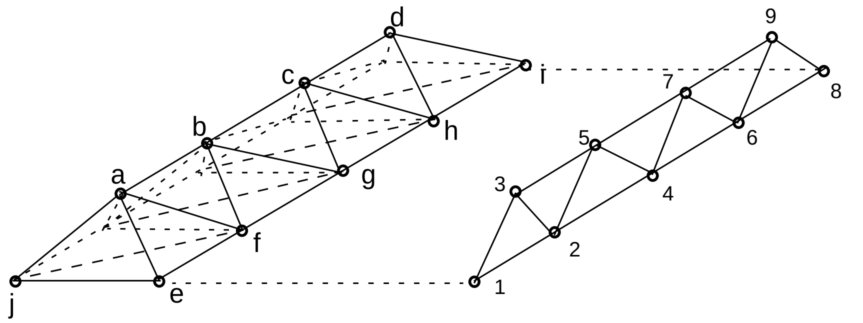
Plot of Member Force against Displacement.



Discussion - Member Force, Experiment & Theory.

Appendix   The 2D Truss Model.

To model the truss shown as a two dimensional planar truss structure, we need to establish an equivalent stiffness for the pairs of diagonal members and for the pairs of bottom members..



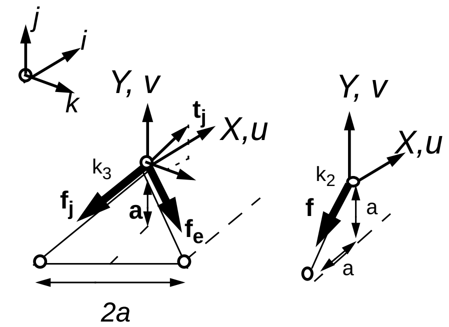
3D Stiffness

We redraw the members of interest and their two-dimensional equivalent. Motion in the “z” direction, perpendicular to the xy plane is taken as zero.

We want to match the elements of the 2 x 2 stiffness matrix of the 2D model with the elements of the 2 x 2 stiffness matrix of the 3D structure.

For the 3D structure, equilibrium of the node gives

$$\boldsymbol{f_j} + \boldsymbol{f_e} + (X\boldsymbol{i} + Y\boldsymbol{j}) = 0$$



3D **2D**

where the member forces are assumed positive in tension and the bold indicates I am treating them as vectors. (**i, j,k** are unit vectors in the x, y and z directions respectively). I show also a unit vector, $\mathbf{t_j}$, lying along the member j but directed oppositely to $\mathbf{f_j}$. The unit vector, using the Pythagorean formula (twice) is

$$\boldsymbol{t}_j = \frac{1}{\sqrt{3}} \cdot \boldsymbol{i} + \frac{1}{\sqrt{3}} \cdot \boldsymbol{j} + \frac{1}{\sqrt{3}} \cdot \boldsymbol{k} \qquad \text{so} \qquad \boldsymbol{f_j} = -\frac{f_{3D}}{\sqrt{3}} \cdot \boldsymbol{i} - \frac{f_{3D}}{\sqrt{3}} \cdot \boldsymbol{j} - \frac{f_{3D}}{\sqrt{3}} \cdot \boldsymbol{k}$$

and

$$\boldsymbol{f_e} = -\frac{f_{3D}}{\sqrt{3}} \cdot \boldsymbol{i} - \frac{f_{3D}}{\sqrt{3}} \cdot \boldsymbol{j} + \frac{f_{3D}}{\sqrt{3}} \cdot \boldsymbol{k}$$

So equilibrium gives

$$-\frac{f_{3D}}{\sqrt{3}} - \frac{f_{3D}}{\sqrt{3}} + X = 0 \qquad \textit{and} \qquad -\frac{f_{3D}}{\sqrt{3}} - \frac{f_{3D}}{\sqrt{3}} + Y = 0$$

We need now the extension of each member for a unit displacement in the x and in the y directions. The extension is found by projecting the unit displacement onto the member; the extensions of member *j* due to a unit displacements in the x and y directions are

$$\delta_j\big|_{u=1} = 1\boldsymbol{i} \bullet \boldsymbol{t}_j = \frac{1}{\sqrt{3}} \qquad \text{and} \qquad \delta_j\big|_{v=1} = 1\boldsymbol{j} \bullet \boldsymbol{t}_j = \frac{1}{\sqrt{3}}$$

while for member *e* , for a unit displacement in the x direction and in the y direction respectively

$$\delta_e\big|_{u=1} = 1\boldsymbol{i} \bullet \boldsymbol{t}_e = \frac{1}{\sqrt{3}} \qquad \text{and} \qquad \delta_e\big|_{v=1} = 1\boldsymbol{j} \bullet \boldsymbol{t}_e = \frac{1}{\sqrt{3}}$$

The extensions, because of symmetry, just as the member forces, are equal, so we replace the member subscript with the notation “3D” and write, for arbitrary displacements *u* and *v*

$$\delta_{3D} = \frac{1}{\sqrt{3}} \cdot u + \frac{1}{\sqrt{3}} \cdot v$$

The member force, f_{3D}, is related to the member’s deformation, δ_{3D}, by

$$f_{3D} = k_{3D} \cdot \delta_{3D} \qquad \text{where} \qquad \boxed{k_{3D} = \frac{A_3 E}{L_3}}$$

so equilibrium in terms of the displacements, u and v, becomes

$$\boxed{\left(\frac{2k_{3D}}{3}\right) \cdot u + \left(\frac{2k_{3D}}{3}\right) \cdot v = X \qquad \textit{and} \qquad \left(\frac{2k_{3D}}{3}\right) \cdot u + \left(\frac{2k_{3D}}{3}\right) \cdot v = Y}$$

2D Stiffness

For the 2D stand-in, we go thru the same analysis; equilibrium in the x and y directions gives

$$-\frac{f_{2D}}{\sqrt{2}} + X = 0 \qquad \textit{and} \qquad -\frac{f_{2D}}{\sqrt{2}} + Y = 0$$

while compatibility of deformation yields

$$\delta_{2D} = \frac{1}{\sqrt{2}} \cdot u + \frac{1}{\sqrt{2}} \cdot v$$

and with the same form for force/deformation relation,

$$f_{2D} = k_{2D} \cdot \delta_{2D} \qquad \text{where} \qquad \boxed{k_{2D} = \frac{A_2 E}{L_2}}$$

we put equilibrium in terms of displacement and obtain

$$\boxed{\left(\frac{k_{2D}}{2}\right) \cdot u + \left(\frac{k_{2D}}{2}\right) \cdot v = X \qquad \textit{and} \qquad \left(\frac{k_{2D}}{2}\right) \cdot u + \left(\frac{k_{2D}}{2}\right) \cdot v = Y}$$

We see that, in order to make the stiffness of the 2D model equivalent to that of the 3D structure we must have

$$\frac{k_{2D}}{2} = \frac{2k_{3D}}{3} \qquad \text{or} \qquad \frac{1}{2} \cdot \frac{A_2 E}{L_2} = \frac{2}{3} \cdot \frac{A_3 E}{L_3} \qquad \text{where} \qquad \begin{aligned} L_2 &= \sqrt{2} \cdot a \\ L_3 &= \sqrt{3} \cdot a \end{aligned}$$

which, when we substitute for the lengths, gives for the ratio of the areas

$$\boxed{\frac{A_2}{A_3} = \frac{4 \cdot \sqrt{2}}{3 \cdot \sqrt{3}} \qquad \text{or} \qquad A_2 = 1.089 \cdot A_3}$$

There remains the question - How to interpret the results for the member force computed in the 2D model? What we need is the ratio of f_{3D} to f_{2D} for the same displacement set u and v. This is obtained from the force/deformation relations, expressing the deformations in terms of the displacements. We have

$$f_{3D} = \frac{k_{3D}}{\sqrt{3}} \cdot (u + v) \qquad \text{and} \qquad f_{2D} = \frac{k_{2D}}{\sqrt{2}} \cdot (u + v)$$

which gives

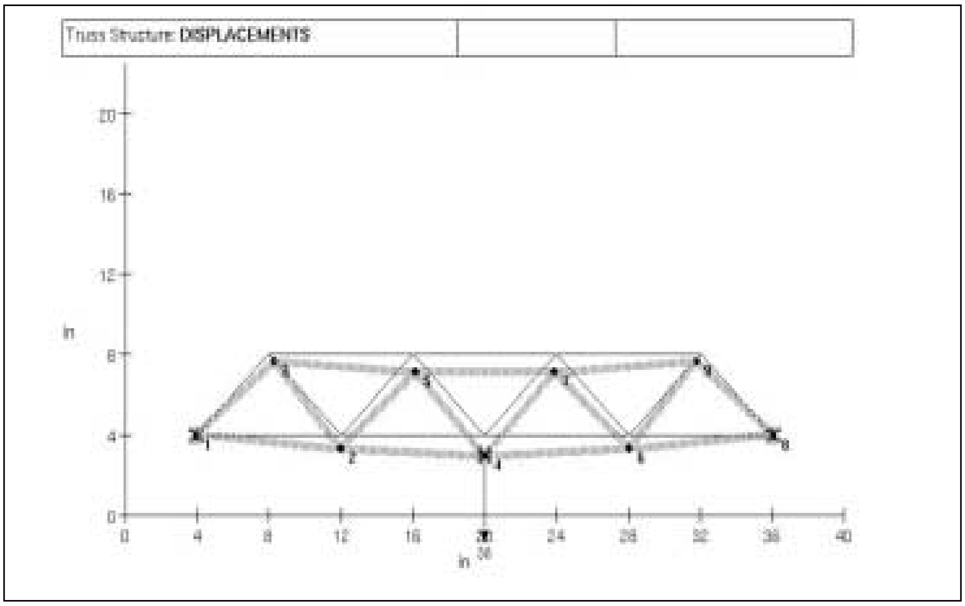
$$\frac{f_{3D}}{f_{2D}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{k_{3D}}{k_{2D}} \qquad \text{or} \qquad f_{3D} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{3}{4} \cdot f_{2D}$$

or, finally

$$\boxed{f_{3D} = 0.612 \cdot f_{2D}}$$

Truss2d Model

Here is the results of a Truss2D model of the three dimensional structure with the top member areas now reduced to .0069 in^2.



NODE	COORDINATES x in	y in	DISPLACEMENTS u in	v in
1	.000E+00	4.000E+00	-5.747E-04	0.000E+00
2	1.200E+01	4.000E+00	-4.310E-04	-6.679E-03
3	8.000E+00	8.000E+00	2.399E-03	-3.412E-03
4	2.000E+01	4.000E+00	0.000E+00	-1.039E-02
5	1.600E+01	8.000E+00	1.199E-03	-8.748E-03
6	2.800E+01	4.000E+00	4.310E-04	-6.679E-03
7	2.400E+01	8.000E+00	-1.199E-03	-8.748E-03
8	3.600E+01	4.000E+00	5.747E-04	0.000E+00
9	3.200E+01	8.000E+00	-2.399E-03	-3.412E-03

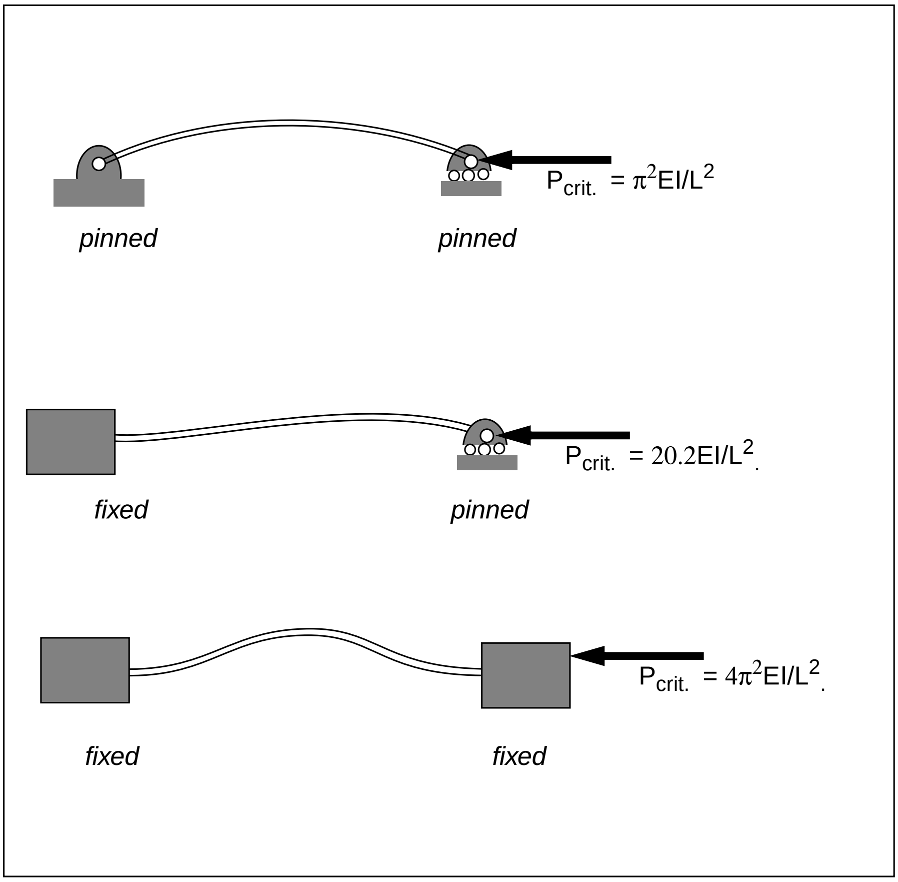
NODE	APPIED LOADS Fx lb	Fy lb	REACTIONS Fx lb	Fy lb
1	0.000E+00	0.000E+00	0.000E+00	1.500E+01
2	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3	0.000E+00	0.000E+00	0.000E+00	0.000E+00
4	0.000E+00	-3.000E+01	0.000E+00	0.000E+00
5	0.000E+00	0.000E+00	0.000E+00	0.000E+00
6	0.000E+00	0.000E+00	0.000E+00	0.000E+00
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8	0.000E+00	0.000E+00	0.000E+00	1.500E+01
9	0.000E+00	0.000E+00	0.000E+00	0.000E+00

i-j	LENGTH in	PROPERTIES A in2	E psi
1-2	8.000E+00	2.880E-02	2.900E+07
1-3	5.657E+00	1.336E-02	2.900E+07
2-3	5.657E+00	1.336E-02	2.900E+07
2-4	8.000E+00	2.880E-02	2.900E+07
2-5	5.657E+00	1.336E-02	2.900E+07
3-5	8.000E+00	6.900E-03	2.900E+07
4-5	5.657E+00	1.336E-02	2.900E+07
4-6	8.000E+00	2.880E-02	2.900E+07
4-7	5.657E+00	1.336E-02	2.900E+07
5-7	8.000E+00	6.900E-03	2.900E+07
6-7	5.657E+00	1.336E-02	2.900E+07
6-8	8.000E+00	2.880E-02	2.900E+07
6-9	5.657E+00	1.336E-02	2.900E+07
7-9	8.000E+00	6.900E-03	2.900E+07
8-9	5.657E+00	1.336E-02	2.900E+07

i-j	TENSILE FORCE lb	TENSILE STRESS psi
========	========	========
1-2	1.500E+01	5.208E+02
1-3	-2.121E+01	-1.588E+03
2-3	2.121E+01	1.588E+03
2-4	4.500E+01	1.562E+03
2-5	-2.121E+01	-1.588E+03
3-5	-3.000E+01	-4.348E+03
4-5	2.121E+01	1.588E+03
4-6	4.500E+01	1.562E+03
4-7	2.121E+01	1.588E+03
5-7	-6.000E+01	-8.696E+03
6-7	-2.121E+01	-1.588E+03
6-8	1.500E+01	5.208E+02
6-9	2.121E+01	1.588E+03
7-9	-3.000E+01	-4.348E+03
8-9	-2.121E+01	-1.588E+03

Euler Buckling Loads

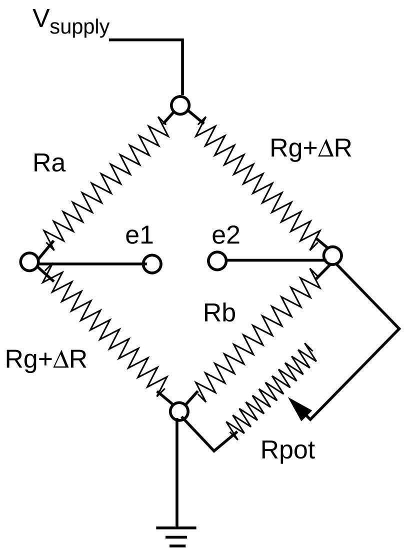
The Euler Buckling Loads for a column with different boundary conditions are shown in the figure.



For a circular cross section of radius *r* $I = \pi r^4/4$.

Strain-gage Circuit Analysis

Two strain gages are fixed to one of the members. They both will measure the extension or contraction along the length. A bridge circuit is used to avoid attempting to measure the small difference of two large numbers and their placement in the bridge also should eliminate the effects of bending - although we anticipate that to be small.



The values of the resistances are:

Ra = 352 +/- 5% ohms

Rb = 561 +/- 5% ohms

Rg = 350 +/-0.2% ohms

Rpot = 1 kohm (max)

The ouput of the bridge as a function of the change in resistance, ΔR,

$$e1 - e2 = (V_{supply}/2)(\Delta R/Rg)$$

The strain as a function of change in resistance is given by

$$\varepsilon = (1/F_{gage})(\Delta R/Rg)$$

where F_{gage} is the "gage factor" stated by the manufacturer[1] to be

F_{gage} =2.07 +/- 0.5%

With these, you can compute the strain in the member, given the voltage difference e1-e2.

The voltage difference e1 - e2 is obtained from your measured values at the op-amp output by dividing by the amplifier gain.

1. BLH Electronics, Inc.