

12.340: Global Warming Science
Problem Set #2: Radiative Transfer and the Greenhouse Effect
Date assigned: Thursday, 1 Mar 2012
Date due: Thursday, 8 Mar 2012

Question 1: Blackbody radiation basics

- a) Start with the expression for Planck's law per unit frequency and convert to Planck's law per unit wavelength.
- b) Derive Wien's Law

$$\lambda_{\max} = \frac{b}{T}$$

$$b = 2.90 \times 10^{-3} \text{ mK}$$

via $\frac{dB(\lambda, T)}{d\lambda} = 0$.

- c) Derive the Stefan-Boltzmann constant, σ in the below equation, in terms of fundamental constants (Planck's constant h , Boltzmann's constant k , and the speed of light c), by integrating the Planck function over the entire wavelength domain,

$$\int_0^{\infty} B_{\lambda} d\lambda = \frac{\sigma T^4}{\pi}$$

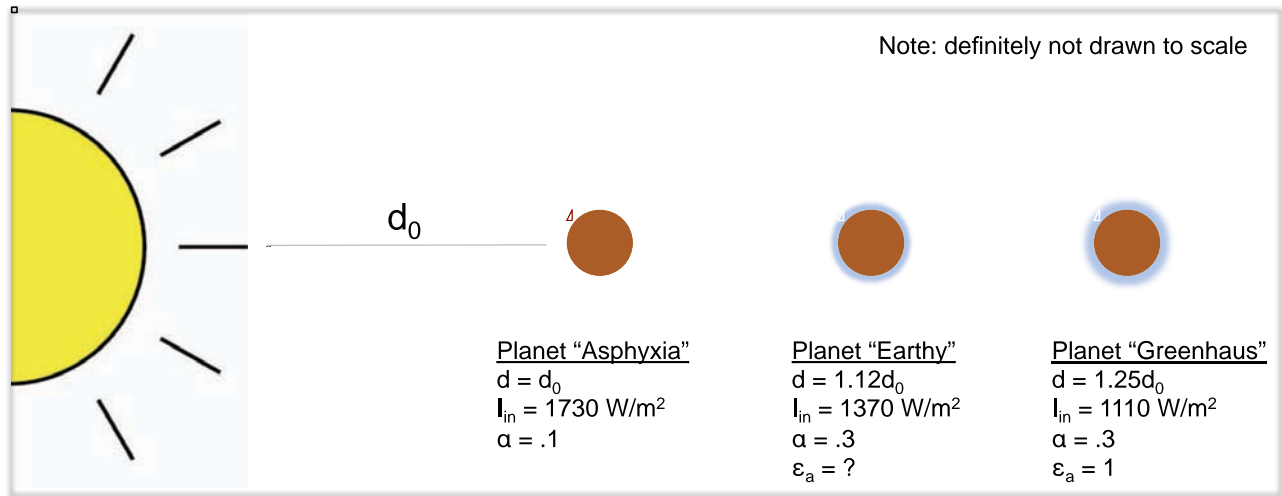
Question 2: The effect of a CO₂ increase on Earth's spectrum

In class we discussed Figure 4.8 in Archer's "Global Warming: Understanding the Forecast". Look at the increased flux in the Earth's spectrum at 550 cm⁻¹ (Fig. 4.8 c).

Is this increased flux due to conservation of energy (same total area under the spectrum curve), or is the increased flux due to a shift of the peak of the blackbody due to the higher temperatures induced by the CO₂ increase? Describe your preferred explanation in a paragraph (possibly by computing blackbodies for relevant temperatures).

Question 3: Simple models for planetary climates

Somewhere in a far off galaxy, there exists a solar system named Tropica that has three planets, each with radius r , orbiting a star.



Planet "Asphyxia" is closest to the sun, at a mean distance of d_0 (where $d_0 \gg r$), and intercepts a plane-parallel beam of incoming solar radiation with an intensity $I_{in} = 1730 \text{ W/m}^2$. It has no atmosphere and a dark surface such that the planetary albedo is .1 .

- a) What is the mean surface temperature on "Asphyxia"? Assume the surface acts like a perfect blackbody.

Planet "Greenhaus" is the farthest from the sun with $I_{in} = 1110 \text{ W/m}^2$ and has a thick atmosphere that is transparent to shortwave (i.e. solar) radiation but is a perfect blackbody for longwave (i.e. infrared) radiation. It also now has clouds and thus a higher albedo of .3 .

- b) Let's model this atmosphere as a single layer that is a perfect blackbody, i.e. it has emissivity of 1. What is the mean surface temperature on "Greenhaus"? Assume the surface acts like a perfect blackbody.

Finally, Planet "Earthy" has an orbit whose radius lies in between the other two planets, such that $I_{in} = 1370 \text{ W/m}^2$. It is very similar to "Greenhaus", except that it has a thinner atmosphere that absorbs only some longwave radiation and only at certain wavelengths.

- c) Let's model this atmosphere again as a single layer only now it is no longer a perfect blackbody. This planet conveniently seems to have properties similar to the real Earth, whose mean surface temperature is approximately 288 K.
- What must the absorptivity of the atmosphere on "Earthy" be to give an Earth-like surface temperature?
 - How might we represent an increase in greenhouse gas concentrations in this model due to, e.g. the emissions caused by the bustling life of "Earthy"?
 - Do you think this "leaky greenhouse" model is a good representation of the real Earth's climate? Why or why not?

Question 4: The two-layer atmosphere model

- a) Problem 3-2 in Archer.
- b) Show that for an N-layer model

$$T_{\text{ground}} = (N+1)^{1/4} T_e$$

where T_e is the emission temperature in the topmost atmosphere layer.

- c) Bonus question: Why might the model breakdown as N gets very large?

Question 5: Greenhouse gases and infrared radiation

For this problem, use the online longwave radiation model at:

<http://geoflop.uchicago.edu/forecast/docs/Projects/modtran.html>

With default settings, this model calculates the spectrum of upwelling longwave (i.e., infrared) radiation near the top of the atmosphere (TOA). The temperature distribution and greenhouse gas concentrations are specified by the user. The output Figure also shows blackbody distributions for temperatures typical of the surface and atmosphere.

- a) Run the model with default settings. Absorption by CO_2 is particularly strong in a band centered at a wavenumber of $\sim 700 \text{ cm}^{-1}$. You will see that the core of this band corresponds to emission from some of the coldest air in the atmosphere ($\sim 220\text{K}$) located high in the troposphere. Further increasing the concentration of CO_2 does not significantly decrease the emission temperature at these wavenumbers; the core of the band is said to be saturated. But the absorption band does become effectively wider as the concentration of CO_2 is increased, so that the outgoing longwave radiation (OLR; $[\text{W}/\text{m}^2]$) continues to decrease. See this broadening for yourself by rerunning the model for CO_2 concentrations of 0.1; 1; 10; 100; 1000 ppm. **Make a plot of OLR versus CO_2 concentration.** It should be clear that OLR is most sensitive to changes in CO_2 at low concentrations, with a roughly logarithmic dependence. **Illustrate the logarithmic dependence by also plotting OLR versus the log of CO_2 concentration.**
- b) Put the CO_2 concentration to a very high value (e.g., 10000 ppm). Why do you think there is an upward spike in the center of the 700 cm^{-1} CO_2 band?
- c) Repeat (a) and (b) for H_2O , choosing a relevant wavenumber, describing if the feature is saturated, and varying concentrations.
- d) Repeat (a) and (b) for CH_4 , choosing a relevant wavenumber, describing if the feature is saturated, and varying concentrations.
- e) Describe the comparative importance of CO_2 , H_2O , and CH_4 as greenhouse gases.

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