Linear Elastic Fracture Mechanics

Assigned Reading:

Atkinson, B.K., Fracture Mechanics of Rock, pp. 534, Academic Press, London UK, 1987. Chapter 1, 2, 4.

Resource reading:

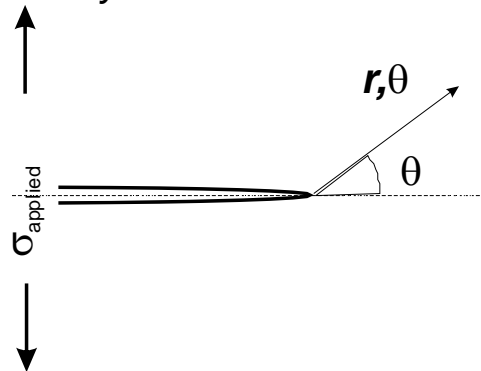Lockner, D.A., Rock failure, in Handbook of Physical Constants, edited by T.A. Ahrens, pp. Sections 3-10, 1995.

Lockner, D.A., Brittle deformation law for Westerly Granite, J. Geophys. Res., 103, 5107-5103, 1998.

Lawn, B.R., and T.R. Wilshaw, *Fracture of brittle solids*, Cambridge University Press, Cambridge, UK, 1975.

Paterson, M.S., *Experimental Rock Deformation - The Brittle Field*, 254 pp., Springer, New York, 1978.

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Stress Intensity Factors

Mode I loading

Local Stress near tip,
a is the half length of the crack:

$$\sigma_{ij} = \sigma_{applied} \frac{\sqrt{a}}{\sqrt{2\pi r}} f^{(ij)}(\theta)$$

where $f^{(ij)}(\theta)$ is of order 1,

the exact form depends on loading mode and geometry (the values of *i, j*).

"General" Equation:

$$\boxed{\sigma_{ij} = \frac{K_i}{\sqrt{2\pi r}} f^{(ij)}(\theta)}$$

where

$$\boxed{K_i = \Psi_i \sigma_{(i).\,applied} (c)^{1/2}}.$$

Ψ_i is a numerical constant the depends on the geometry of the loading and the stresses applied (i.e. shear, normal), crack geometry, and edge effects; c is the half-dimension characteristic of the crack geometry.

Notice that i.) the near crack stress is multiplied by the flaw by an amount proportional to the square root of the half-length.
ii.) the stress is unbounded when r=0. The energy density within the near-crack region depends on the applied stress and the geometry.
iii.) the K factors depend only on the external loading conditions and the geometry.
iv.) the open crack will have a parabolic cross-section, with the maximum opening being proportional to the applied stress:

$$u(X) = K/E(8X/\pi)^{1/2}$$ *where X is the distance back from the crack tip.*

Critical stress intensity factor:

It is unreasonable to suppose that stresses can indeed become unbounded in the material near the crack tip. At some point the material no longer behaves as an elastic material and the near tip stresses are relaxed. It seems reasonable to assume that if some critical energy density is exceeded, the crack will extend. Griffith considered the thermodynamics of extending cracks and reasoned that the mechanical energy associated with the work being done by displacements at the far field surfaces must be balanced by the energy associated with the surfaces being formed and the energy stored in the elastic field around the crack. If the crack extends irreversibly, there will also be energy dissipated as heat or in other irreversible processes. The critical strain energy release rate is G_c, i.e. twice the energy released per unit area fracture surface being formed. The critical energy release rate must be achieved to extend the crack; it can be shown that there must be a critical stress intensity factor achieved. This K_{iC}, critical stress intensity factor or fracture toughness, is a material parameter only. When plane strain conditions and mode I loading are imposed:
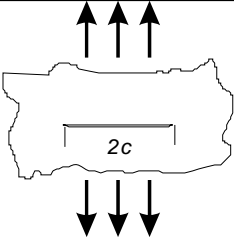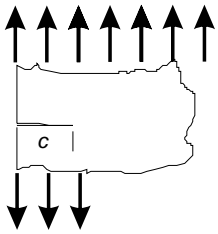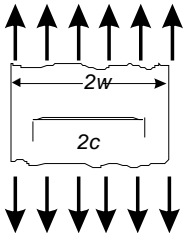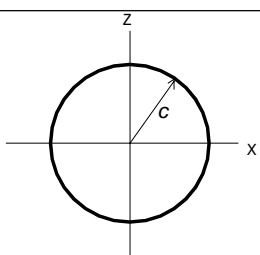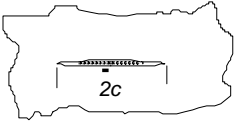
$$G_c = 2Q_f = K_{Ic}^2(1-v^2)/E$$

Note:Imagine that the local stress is proportional to K_{iC}, and that the local strain is proportional to K_{iC}/E. The product is proportional to the elastic energy stored.
The critical stress intensity is a material parameter.
A crack will extend when the applied stress causes the stress intensity to exceed the critical stress intensity:For example for a 2d crack in an infinite medium $$K_{Ic} = \sigma_{applied}\sqrt{\pi a}$$

Configuration	Description	ψ
	Double-ended straight crack embedded	$\pi^{1/2}$
	Edge crack	$1.2\pi^{1/2}$
	Double-ended straight crack embedded in finite width	$\left[\dfrac{2w}{c}\tan\left(\dfrac{\pi c}{2w}\right)\right]^{1/2}$
	Penny-shaped crack with radius c, uniform tension along y axis	$2/\pi^{1/2}$
	Straight and penny shaped crack in infinite medium with internal pressure	$\pi^{1/2}$ (straight) $2/\pi^{1/2}$ (penny)