12.864 Inference from Data and Models 4 April 2005
Problem Set No. 4 Due: 20 April 2005

1. A physical system is believed governed by the difference equation

$$y(n+1) - 2y(n) + y(n-1) = p(n)$$

where $\tilde{y}(0) = 0 \pm 1, \tilde{y}(-1) = 0 \pm 2$. $\left\langle p(n)^2 \right\rangle = 0.2$.

(a) Put it into canonical form, and let the state vector be called $\mathbf{x}(n)$.

(b) Time-step the system until $n = 10$ and plot the value of y from 0 to 10. Use $p(n) = [0.3413, 0.0738, -0.0898, -0.3509, 0.3702, -0.1159, -0.6736, 0.4609, 0.0901, -0.5542]$, $0 \leq n \leq 9$.

(c) At $n = 5$, you have a measurement $y(5) - y(4) = -0.05 \pm 0.1$. Using a Kalman filter of your own coding, make an estimate of $y(n)$ from $n = 0$ to 10 and calculate an error bar on the estimate.

(Explanation: $p(n)$ is given to you so that you can compute a time trajectory knowing what the true forcing is. It's only for background information. In running the Kalman filter, you do not know the values $p(n)$—which correspond to $\mathbf{\Gamma u}(t)$. $\mathbf{q}(t) = 0$ here. Information that is missing is commonly set to zero.)

2. For the physical situation of Problem 1 (model, initial conditions with error) and the observation at $t = 5$, make an improved estimate of $\tilde{y}(0)$ by running the model and Kalman filter backwards and time. (But do *not* use the result of the Kalman filter from Problem 1.)

3. From the result in Problem 1, and using the RTS algorithm, make an improved estimate of $\tilde{y}(0)$ in problem 1, and estimate $p(n)$ with error bar.

4. Using the model, initial conditions, and observations of problem 1, re-solve for $\tilde{y}(n)$ using the method of Lagrange multipliers (adjoint method).