

14.02 Principles of Macroeconomics

Spring 2014

Problem Set 2

Due: March 7, 2014

Bank Run Model (40 points)

This exercise leads you to go through the derivation of the bank run model.

There are 3 periods. At $t = 0$, all agents start with 1 unit of endowment and can invest in 2 different technologies:

1. A short term technology that delivers 1 unit of output at $t = 1$ for each unit of output invested at $t = 0$.

2. A long term technology that delivers $R > 1$ units of output at $t = 2$ for each unit of output invested at $t = 0$. However, if this technology is liquidated at $t = 1$ it delivers $L < 1$ units of output.

There are two types of agents (patient and impatient) differing in their consumption preference. Patient agents are indifferent between consuming at $t = 1$ or at $t = 2$. Impatient agents only consume at $t = 1$.

To simplify the setting, we further assume – differently from the DD model – that it is impossible to store goods between $t = 1$ and $t = 2$. Hence, if a patient agent receives some goods in $t = 1$, she must consume them readily in $t = 1$.

a). Consider a simple case: if agents know their type before investing, what is the optimal investment strategy for patient and impatient types? (5 points)

Answer:

Patient agent invests 1 unit in long-term technology, and impatient agent invests 1 unit in short-term technology.

Now, consider the general case presented in the lecture note. In all questions below, we assume that agents learn their type only at $t = 1$. That is, agents do not know their type at $t = 0$ when making investment decisions. However, they know that the probability of being patient is π , and the probability of being impatient is $1 - \pi$. Agents cannot trade with each other.

b). Let α be the amount of endowment invested in the long-term technology. What is the consumption at $t = 1$ and $t = 2$ for patient and impatient types? (5 points)

Answer:

Impatient agent only values consumption at $t = 1$, so they will liquidate long-term investment and consume $(1 - \alpha) + \alpha L$.

Patient agent consumes $1 - \alpha$ at $t = 1$, and αR at $t = 2$.

c). If the goal is to maximize expected consumption, what is the optimal value for α ? (5 points)

Answer:

Total consumption for impatient agent is $(1 - \alpha) + \alpha L$, while total consumption for patient agent is $(1 - \alpha) + \alpha R$.

Agents do not know their type when making investment, thereby their expected consumption is,

$$\pi[(1 - \alpha) + \alpha R] + (1 - \pi)[(1 - \alpha) + \alpha L]$$

which is linear in α .

This suggests the optimal decision depends on the value of π ,

If $\pi > \frac{1-L}{R-L}$, $\alpha = 1$, invest all in long-term technology.

If $\pi < \frac{1-L}{R-L}$, $\alpha = 0$, invest all in short-term technology.

If $\pi = \frac{1-L}{R-L}$, agent is indifferent.

Intuitively, if probability of being patient is larger, agent prefers to invest in the long-term technology.

d). Suppose agents can trade bond contract at $t = 1$ (see lecture notes). The price of the bond is p . If trade occurs, it must be $\frac{L}{R} \leq p \leq 1$, why? (7 points)

Answer:

Note that only the patient agents can possibly have incentive to purchase the contract, because impatient ones do not value consumption at $t = 2$.

If $p > 1$, patient agent has no incentive to purchase the contract, because they are indifferent between consumption at $t = 1$ or $t = 2$. But he would be worse off by paying more at $t = 1$ than what he can get at $t = 2$.

If $p < \frac{L}{R}$, impatient agent has no incentive to sell the contract, as they can just liquidate the long-term investment at L .

e). Calculate the equilibrium p and α that maximize expected consumption. Show that expected consumption in the trading equilibrium is higher than the autarky case. (10 points)

Answer:

Assume $\frac{L}{R} \leq p \leq 1$, so agents do have incentive to trade at $t = 1$ (You can verify that the equilibrium p satisfies this condition).

For impatient agent, the total consumption is $1 - \alpha + pR\alpha$. For patient agent, the total consumption is $\frac{1-\alpha}{p} + R\alpha$.

Therefore, expected consumption is

$$\pi\left(\frac{1-\alpha}{p} + R\alpha\right) + (1-\pi)(1-\alpha + pR\alpha)$$

which is linear in α (again).

If $p > \frac{1}{R}$, all agents choose $\alpha = 1$, supply of bonds = $R(1 - \pi) >$ demand for bonds = 0.

If $p < \frac{1}{R}$, all agents choose $\alpha = 0$, supply of bonds = 0 < demand for bonds = $\frac{\pi}{p}$.

Hence, only when $p = \frac{1}{R}$, there exists an equilibrium in the bonds market, which pins down $\alpha = \pi$.

f). If agents cannot trade, but all agents delegate bank to make investment. Show that the bank can achieve the trading equilibrium. (8 points)

Answer:

The bank can do the following to achieve the equilibrium in the trading environment.

At $t = 0$, agents deposit all their endowments in the bank. Bank invests π in long-term assets.

At $t = 1$, Bank gets $1 - \pi$ from its investment in short-term technology. Impatient agents withdraw their money and get 1, the total amount of withdrawing is $1 - \pi$.

At $t = 2$, Bank gets πR from its investment in long-term technology. Patient agents withdraw their money and get R , the total amount of withdrawing is πR .

g). (Bonus) If the bank's goal is to maximize agent's expected consumption, what is the optimal α ? Why this is not achievable in a pure trading economy. (10 points)

Answer:

The optimal $\alpha = 1$, that is, bank invests only in the long-term technology. The allocation is as follows: impatient agent receives nothing, while patient agent receives $\frac{R}{\pi}$. The expected consumption is $(1 - \pi)0 + \pi \frac{R}{\pi} = R > 1$ (1 is the expected consumption in the trading equilibrium).

Essentially this result relies on the assumption that patient agent values consumption indifferently between $t = 1$ and $t = 2$ (risk neutrality). In this case, the optimal rule is to invest everything in the long-term technology, which provides a higher return, and not liquidate it at $t = 1$. This implies that the agent is playing a very risky lottery at $t = 0$: getting nothing if being impatient, and getting more if being patient. All agents are willing to play this lottery precisely because they are risk neutral.

In the lecture notes, the assumption is agent is risk averse, so let the impatient guy receive zero consumption will never be an optimal solution.

In a pure trading economy, this is not possible because the agent ends up being impatient has no incentive to forgo his consumption (unless there is strict legal coercion).

1 IS-LM Model (30 points)

a). Suppose that the economy has no money market and that I, G, T are given. Let $C = c_0 + c_1(Y - T)$. Derive equilibrium output and the government spending multiplier. (8 points)

Answer:

Spending is given by $Z = C + I + G = c_0 + c_1(Y - T) + I + G$. Equilibrium in the goods market will be given by

$$Y = c_0 + c_1(Y - T) + I + G$$

that yields

$$Y = \frac{1}{1 - c_1}[c_0 - c_1T + I + G]$$

The spending multiplier is given by

$$\frac{dG}{dY} = \frac{1}{1 - c_1}$$

b). Assume now that investment is $I = b_0 + b_1Y - b_2i$. Suppose that the interest rate is exogenously fixed. Derive equilibrium output and the government spending multiplier. How does this spending multiplier compare to the one you found in part a)? (8 points)

Answer:

Now,

$$Z = C + I + G = c_0 + c_1(Y - T) + b_0 + b_1Y - b_2i + G$$

so

$$Y = \frac{1}{1 - c_1 - b_1}[b_0 + c_0 - c_1T - b_2i + G]$$

and

$$\frac{dG}{dY} = \frac{1}{1 - b_1 - c_1}$$

It is bigger than the one we found in part one. The reason is that investment depends positively on output, so we have an additional multiplier effect.

c). Introduce a money market in which $MP = d_1Y - d_2i$. Derive equilibrium output. Derive the government multiplier. Is it bigger or smaller than the one you computed in part b)? Provide some intuition for this result. (14 points)

Answer:

To derive Equilibrium output, just solve

$$Y = \frac{1}{1 - c_1 - b_1}[b_0 + c_0 - c_1T - b_2i + G]$$

$$\frac{M}{P} = d_1Y - d_2i$$

To simplify the algebra denote

$$A = \frac{b_0 + c_0 - c_1T + G}{1 - c_1 - b_1}$$

$$B = \frac{b_2}{1 - c_1 - b_1}$$

So,

$$Y = A - Bi$$

Solving, we get that

$$Y = \frac{Ad_2}{Bd_1 + d_2} + \frac{M}{P} \frac{B}{Bd_1 + d_2}$$

$$i = \frac{Ad_1}{Bd_1 + d_2} - \frac{M}{P} \frac{1}{Bd_1 + d_2}$$

The multiplier is given by

$$\begin{aligned} \frac{dY}{dG} &= \frac{d_2}{Bd_1 + d_2} \frac{dA}{dG} \\ &= \frac{1}{\frac{b_2 d_1}{d_2} + (1 - c_1 - b_1)} \end{aligned}$$

This multiplier is lower. It is intuitive because part of the increase in government spending will crowd out investment due to higher interest rates.

2 The Amplification Mechanism (30 points)

To answer this question, you need to read page 74 – 83 of the lecture notes. Assume $f'(\bar{k}) < a$, namely, the marginal return facing landlords is lower than the marginal return facing entrepreneurs when all capital is invested by landlords.

a). Consider the benchmark case with no frictions, so entrepreneurs do not face collateral constraint (the equation on page 77). What is the equilibrium price? (10 points)

Answer:

The condition $f'(\bar{k}) < a$ guarantees that in a frictionless financial market, landlord is willing to sell capital to the entrepreneur. Let the equilibrium price of capital is q . The entrepreneur can borrow 1 unit of loan from landlord with gross interest rate R , purchase $\frac{1}{q}$ amount of capital, and invest in a linear technology with marginal return a .

The entrepreneur is willing to do so only if the gross return $\frac{1}{q}a + \frac{1}{q}q \geq R$. Notice that inequality cannot hold, since otherwise entrepreneur would demand infinite amount of capital.

Hence, in equilibrium

$$q = \frac{a}{R - 1}$$

Note that the above argument still applies if entrepreneur uses his own funds, then R is the opportunity cost of investment.

b). Consider the general case with collateral constraint. If entrepreneur's personal wealth is w_t at time t , what is the maximum amount of capital he can purchase? (10 points)

Answer:

The borrowing constraint restricts the maximum amount of loan. By borrowing $\frac{q_{t+1}k_t^E}{R}$, the entrepreneur can use $w_t + \frac{q_{t+1}k_t^E}{R}$ to purchase capital at price q_t . Hence, he can purchase at most,

$$k_t^E = \frac{w_t + \frac{q_{t+1}k_t^E}{R}}{q_t}$$

Solve the equation,

$$k_t^E = \frac{w_t}{q_t - \frac{q_{t+1}}{R}}$$

c). What happens to capital invested by the entrepreneur if w is reduced by Δw ? (10 points)

Answer:

When w is reduced by Δw , capital investment is reduced by $\frac{1}{q_t - \frac{q_{t+1}}{R}} \Delta w > \Delta w$. The reduction in investment is larger than the decrease in w because entrepreneur is leveraging.

Note that a fall in w will also reduce both q_t and q_{t+1} , which amplifies the shock.

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