

Problem Set 4
14.003/14.03, Fall 2010

1 Comparative Advantage

Two countries, Home and Foreign, use one factor, labor, to produce two goods, shoes and cars. The Home country can produce shoes with one unit of labor and cars with two units of labor. The Foreign country can produce shoes with 3 units of labor and cars with 4 units of labor. Home country is endowed with a labor force of 200 units, while Foreign country is endowed with a labor force of 100 units. Preferences are the same in the two countries and are described by the following utility function:

$$U(S, C) = \ln S + \ln C$$

1. Which country has an absolute advantage in producing shoes? Which country has an absolute advantage in producing cars?
2. Which country has a comparative advantage in producing shoes? Explain in words why this is the relevant information to determine the pattern of production in an economy where the two countries are allowed to trade.
3. Draw for both countries a graph with the production possibility frontier and the graphical solution to the maximization problem.
4. What are production and consumption in the Home country when in autarky? What is the price ratio $\frac{P_S}{P_C}$ in the autarky equilibrium?
5. What are production and consumption in the Foreign country when in autarky? What is the price ratio $\frac{P_S}{P_C}$?
6. The countries can now trade with each other. Assume the world price ratio is $\frac{P_S}{P_C} = \frac{5}{8}$. How much of each good does each country produce and consume at these prices? Is this an equilibrium? Explain. *Hint: You should think of the country's problem as two separate decisions. Given prices, each country chooses what to produce in order to maximize profits (income). Given income and prices each country chooses what to consume to maximize utility.*
7. The equilibrium world price ratio is $\frac{P_S}{P_C} = \frac{1}{2}$. How much of each good does each country produce and consume at these prices? Keep in mind that in equilibrium all markets should clear. Is total welfare higher now than under autarky?
8. Explain what is the source of gains from trade in this problem.

2 Trade in the Edgeworth Box

Ann and Bob live in an economy closed to international trade. Suppose Ann has utility $u_A(X, Y) = \min(X_A, Y_A)$, while Bob's utility is $u_B(X, Y) = X_B + Y_B$. Ann's initial endowment is 10 units of X and 0 units of Y , while Bob has 0 units of X and 8 units of Y .

1. Draw the Edgeworth box diagram, draw the initial allocation of resources, and draw the indifference curves for Ann and Bob passing through the endowment point.
2. What are Ann and Bob's uncompensated (Marshallian) demand functions for goods X and Y , $D_x(p_x, p_y, I)$ and $D_y(p_x, p_y, I)$? Briefly explain. [Note: No Lagrangians or indirect utility functions are needed to solve this problem. Just think about the shape of their indifference curves.]
3. Given these utility functions, find the set of Pareto efficient allocations, i.e., the contract curve. Draw the contract curve in the Edgeworth Box [Assume that Ann and Bob can trade fractions; hence, you do not have to worry about integer constraints].
4. Find the autarky general equilibrium outcomes for $\frac{p_x}{p_y}$, X_A , Y_A , U_A , X_B , Y_B and U_B when Ann and Bob are allowed to engage in voluntary trade with one another. *Label this point as A in the Edgeworth box you drew in part (1).* [Hint: Because of the shape of Ann's indifference curves, you will not be able to find a point where both indifference curves are tangent to the price line. The key is to remember that at the equilibrium price ratio, both consumers must choose the same point on the Contract Curve - otherwise, their demands are not compatible.]
5. Suppose the economy now opens to international trade. The international price ratio, $\left(\frac{p_x}{p_y}\right)_T = \frac{1}{4}$. Starting from the same endowment as before, what will Ann consume and what will Bob consume? Are they better or worse off than before, when the economy was closed to international trade?
6. Assuming that the economy stays open to trade, calculate the following:
 - How much money would we have to give to Ann or take away from Ann to make her just as well off as she was under autarky?
 - How much money would we have to give to Bob or take away from Bob to make him just as well off as he was under autarky?
 - What is the total surplus from opening to trade (that is, the sum of Ann and Bob's monetary gains from trade)?

3 Externalities in Production

Two steel mills, A and B, are located upstream from a fishery. The steel mills produce steel from labor according to the following production functions:

$$\begin{aligned}s_A &= f_A(l) = \sqrt{l_A} \\ s_B &= f_B(l) = \sqrt{l_B}\end{aligned}$$

for A and B respectively. If s units of steel are produced, the firms also unavoidably produce waste that they dump into the river. They produce waste according to:

$$\begin{aligned}h_A(s) &= \frac{1}{2}s^2 \\ h_B(s) &= s^2\end{aligned}$$

The fishery produces f units of fish according to the amount of labor it employs and the total amount of pollution in the water:

$$f = g(l_f, h) = 2\sqrt{l_f} - h$$

where $h = h_A + h_B$ is the total level of pollution in the water caused by the two steel mills.

The price of steel is $p = 2$, the price of fish is $q = 2$ and wages are $w = 1$.

Note: The numerical answers for this question are important, so you should try to approximate where needed.

1. What is the profit maximizing choice of output for each steel mill and the fishery? How much pollution does each factory emit? What is total welfare? Is this equilibrium socially optimal? Explain why or why not.
2. Assume the fishery buys out the two steel mills. What is the profit maximizing choice of output in this case? What is the level of pollution for each plant? Compare to the level of pollution in part (1). What is total welfare? Is this outcome socially optimal? Explain why or why not.
3. The government can levy a Pigouvian *tax per unit of pollution*, the same tax for both steel mills, to achieve the socially optimal total level of pollution. At what level should the government set this tax? What are output and profits at each plant? Is the final allocation Pareto efficient?
4. Assume instead that the government does not observe the actual level of pollution for each steel mill. Hence, it can only levy a *tax on output*. The government imposes a uniform linear tax per unit of output for the steel mills. At what level should they set the tax to achieve the socially optimal total level of pollution? How is pollution distributed across the two plants? What is the total level of output? What are output and profits at each plant? Is the final allocation socially optimal? Compare to your answer in part (3). Explain.

5. Recognizing the existence of the externality, the government caps total pollution from the steel mills at $\frac{17}{72}$ units. The government assigns Steel Mill A pollution permits up to the pollution cap of $\frac{17}{72}$. This means that Steel Mill A can produce up to $\frac{17}{72}$ units of pollution, or it can sell any share of this allotment to Steel Mill B or the fishery. If it wants to sell these pollution permits, Steel Mill A announces a market price per permit, and sells the number of permits that Steel Mill B and the fishery demand at this price. Each steel mill needs to have as many permits as they emit in total pollution. If the fishery buys permits then it can decrease the total pollution in the market. What will be the equilibrium price for permits? What are the output and profits at each plant? Is the final allocation socially optimal? Explain. [Hint: You should find the price that clears the market. Assume the steel mills and the fishery take the price as given when optimizing, then find the permit price such that total quantity of permits demanded is equal to the cap].
6. Assume that instead the government gives all the pollution permits to the fishery. The fishery can trade with the two steel mills as above, or keep some of the permits if it wants to decrease the level of pollution in the economy. At what price will the fishery sell the permits, how many permits will it sell, what is the equilibrium allocation of permits, and what are output and profits at each plant?
7. Explain the similarities and differences between your answers to (5) and (6) in light of the Coase Theorem.
8. Without doing any math, what do you think would happen if the government sets the pollution cap at a level higher than $\frac{17}{72}$. Who buys the extra pollution permits? Can the market equilibrium be socially efficient in this case?

4 Instrumental Variables

You are interested in the labor-supply consequences of childbearing. In particular, you want to measure the effect of having more than 2 children (compared to having two children) on labor market participation of mothers, measured as the number of weeks in the labor force during the past year. You have data on number of weeks worked, number of children and dates of birth for the children.

1. You compare the average number of weeks worked last year for those with 2 children and those with more than two children and find that these means differ by approximately 9 weeks. More precisely, $E[\text{weeks}|c > 2] - E[\text{weeks}|c = 2] = -8.97$. Explain why this may not be a causal relationship. If this is not the causal effect of interest, how would you expect it to be biased and why? [Be specific and precise].

Joshua Angrist and William Evans used variation in fertility induced by whether the second and third children were twins as an instrument for family size. In particular, parents of

twins will automatically have an additional child, while parents with a single birth for their second child are less likely to go on and have another child. Whether or not you have twins is virtually randomly assigned.

2. Angrist and Evans propose using 'an indicator for whether the second and third children were twins' as an instrument for whether parents have more than 2 children. Call Z the instrument, where $Z = 1$ means that the second and third children were twins, and $Z = 0$ means that the second birth was a single child.

(a) What are the two conditions necessary for Z to be a valid instrumental variable for assessing the causal effect of having more than 2 children on number of weeks worked? Be precise.

(b) Which of these conditions is testable, and how would you test it?

3. You have compiled the following data:

	Twins	Not Twins
	$Z=1$	$Z=0$
$E[\text{weeks}]$	25.2	27.1
$E[\text{more than 2 children}]$	1.0	.4

(a) What is the estimate of the causal effect of having twins on number of weeks worked?

(b) What is the estimate of the causal effect of having twins on having more than 2 children?

(c) What is the instrumental variables estimate of the causal effect of having more than 2 children on the number of weeks worked?

Demographers have found that mothers of twins are more educated on average. It is well known that twins are more likely for older women, so this probably reflects more childbearing at older ages among more educated women.

4. Assume that the demographers are right: mothers that have twins are more educated and are likely to have stronger labor market attachment. How should this affect your interpretation of the findings in part (3)? Would this cause you to over-estimate or under-estimate the causal effect of having more than 2 children on earnings? Explain.

Note: This question is based on the paper "Children and their parents' labor supply: Evidence from exogenous variation in family size" by Joshua Angrist and William Evans, AER, 1998.

5 Short Questions

1. In the Hsieh and Moretti real estate brokers example, imagine a setting where the number of brokers in each city is fixed (that is, it does not rise or fall with house prices). Would this make the dissipative externality problem more or less severe, and why?

2. Countries A and B produce coffee and sushi. Both countries are closed to international trade. Country A is poor. Citizens must work 1 full day of work to buy a cup of coffee and 4 full days to buy a plate of sushi. Country B is rich. Citizens must work 15 minutes to buy a cup of coffee and must work 1 hour to buy a plate of sushi. How would free trade between countries A and B affect welfare in each country, and why?

For the following statement, say whether it is true/false/uncertain and give a short explanation.

3. An economy is populated by individuals who have the same preferences. We can conclude that there is no welfare gain from allowing them to trade.

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