

## 14.05 Intermediate Macroeconomics

### Problem set 3

Due: March 22nd

**Problem 1.** Consider the Planner's problem in the Ramsey problem. Suppose there are only two periods and suppose further that the utility is logarithmic, that is,  $u(c) = \log(c)$ ; output in period  $t$  is given by  $y_t = A_t k_t^\alpha$ , for some  $\alpha \in (0, 1)$ , and depreciation  $\delta = 0$ . The Ramsey problem is therefore given by

$$\begin{aligned} & \max \log(c_0) + \beta \log(c_1) \\ st: \quad & c_0 + k_1 = A_0 k_0^\alpha \\ & c_1 = A_1 k_1^\alpha \end{aligned}$$

- (i) Draw the set of feasible consumption bundles  $(c_0, c_1)$ . Next, draw the planner's indifference curves and identify the optimum.
- (ii) Consider an increase in  $A_1$ . What changes in the graph? How does the increase in  $A_1$  affect  $c_0, c_1$ , and  $k_1$ ? Give an intuition and explain the relevant income and substitution effects.
- (iii) What is the effect of an increase in  $A_0$ ?

**Problem 2.** Consider the competitive equilibrium in the Ramsey model (where the saving rate is endogenous). The rates of technological progress and population growth are zero ( $n = g = 0$ ). The technology is given by  $y = f(k) = Ak^\alpha$  for some  $\alpha \in (0, 1)$ , the depreciation rate is  $\delta \in (0, 1)$ , and the per-period utility is  $u(c) = \frac{c}{1-\theta}^{1-\frac{1}{\theta}}$ , for some  $\theta > 0$ .

The country under consideration features an inefficient law enforcement system and an extensive degree of corruption, fraud, theft, and the like. As a result, when an individual agent decides how much invest, he/she expects a fraction  $\eta$  of his/her capital returns to be confiscated by other agents in the economy. (Note that this fraction of capital returns does not disappear from the economy; it simply becomes part of the income and consumption of other agents.)

Now suppose that, starting from the steady state corresponding to the above conditions, the country undertakes a reform of its legal and law enforcement system. The reform

reduces the effective rate of confiscation from  $\eta$  to  $\eta^{new}$ , where  $0 < \eta^{new} < \eta$ . The reform is unanticipated and the aforementioned change happens immediately and is expect to last for ever.

(i) Use the phase diagram to describe the initial steady state (prior to reform) and the new steady state (after the reform). What is the impact of the reform on the long-run levels of capital, output, consumption wages, and the interest rate? Explain the intuition behind these effects.

(ii) Use the phase diagram to describe the transition of the economy from its initial position to the new steady state. What happens to capital, output and consumption at the exact moment that the reform is initiated? What happens to these macroeconomic variables after this initial impact effect?

**Problem 3.** Consider the competitive equilibrium in the Ramsey model (where the saving rate is endogenous). This exercise asks you to work out the case of productive government spending. The technology is given by  $y = f(k) = Ak^\alpha g^\beta$  for some  $\alpha, \beta \in (0, 1)$ , the depreciation rate is  $\delta \in (0, 1)$ , and the per-period utility is  $u(c) = \frac{c}{1-\theta}^{1-\frac{1}{\theta}}$ , for some  $\theta > 0$ . The government finances this spending  $g$  with a distortionary tax  $\tau$  on capital income.

(i) Find the conditions that characterize the steady state and  $\dot{c}$  and  $\dot{k}$ . Use the phase diagram to describe this economy.

(ii) Now suppose that, starting from the steady state corresponding to the above conditions, the government undertakes a reform which makes his government spending more productive, i.e.  $f_g$  increases. The reform is unanticipated and the aforementioned change happens immediately and is expect to last for ever. Use the phase diagram to describe the initial steady state (prior to reform) and the new steady state (after the reform). What is the impact of the reform on the long-run levels of capital, output, consumption wages, and the interest rate? Explain the intuition behind these effects.

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