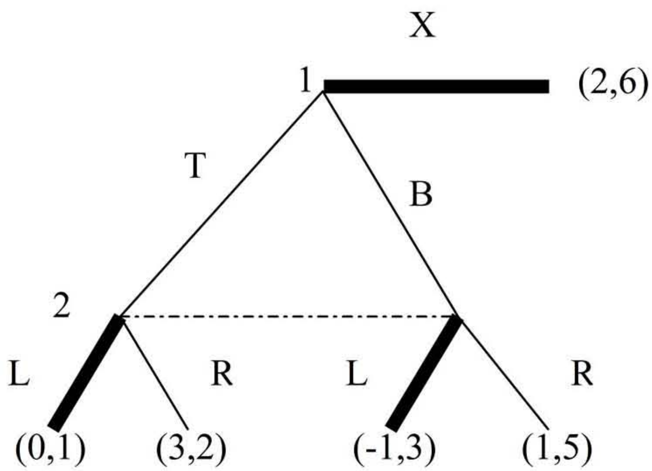
Lectures 20-23
Dynamic Games with Incomplete Information

14.12 Game Theory
Muhamet Yildiz

Road Map

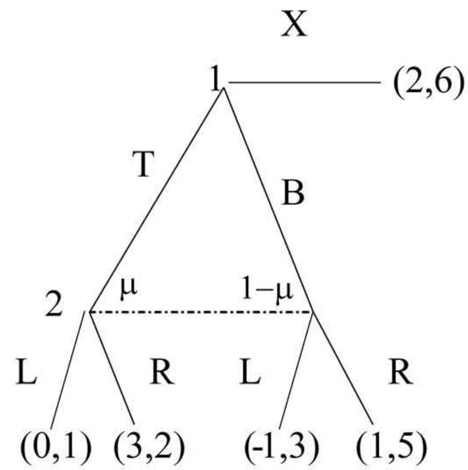
1. Sequential Rationality
2. Sequential Equilibrium
3. Economic Applications
    1. Sequential Bargaining with incomplete information
    2. Reputation

What is wrong with this equilibrium?



Beliefs

- Beliefs of an agent at a given information set is a probability distribution on the information set.
- For each information set, we must specify the beliefs of the agent who moves at that information set.



Assessment

- An assessment is a pair (σ,μ) where
  - σ is a strategy profile and
  - μ is a belief system: μ(.|*I*) is a probability distribution on *I* for each information set *I*.

Sequential Rationality

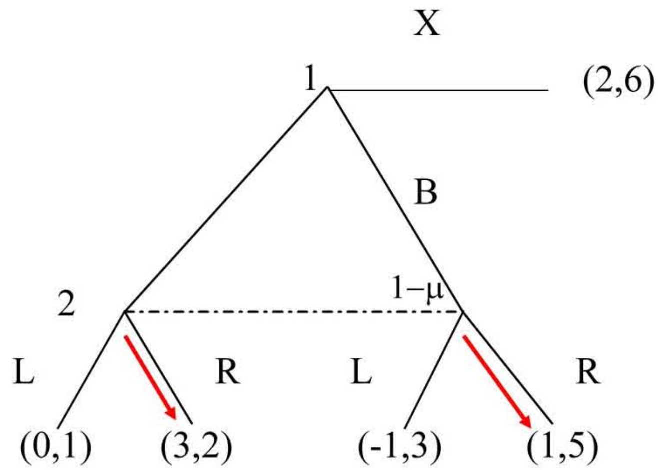
- An assessment (σ,μ) is sequentially rational if

  σ_i is a best reply to σ_{-i} given μ(.|*I*)

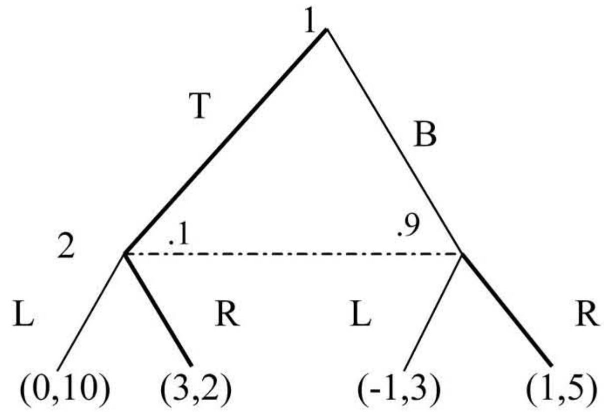
for each player *i*, at each information set *I* player *i* moves.

- That is, σ_i maximizes his expected payoff given μ(.|*I*) and given others stick to to their strategies in the continuation game.

Sequential Rationality implies



Example



Consistency

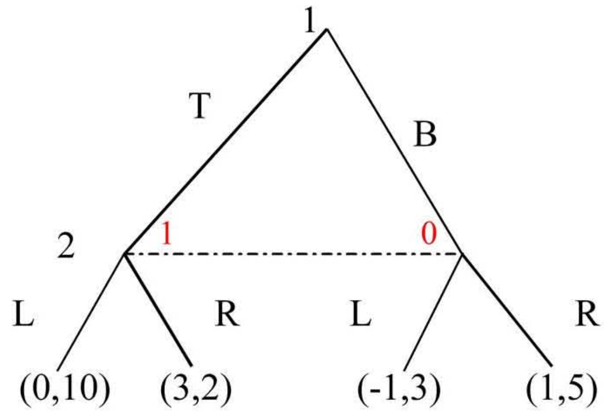
(σ,μ) is consistent if **there is** a sequence

$$(\sigma^m,\mu^m) \to (\sigma,\mu)$$

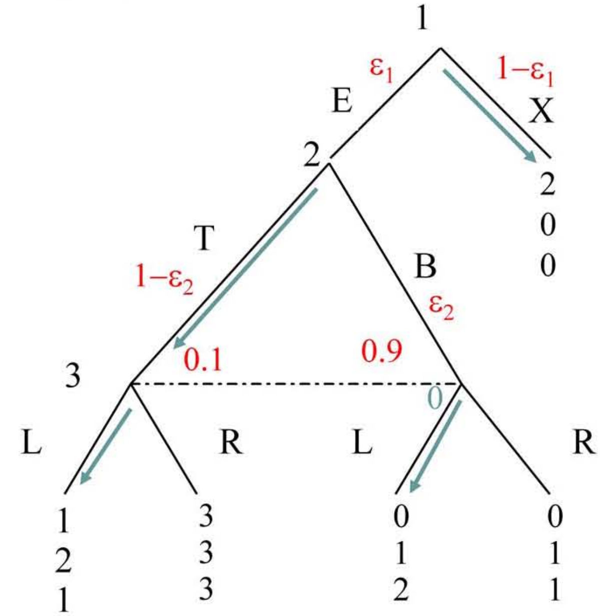
where

- σ^m is "completely mixed" and
- μ^m is computed from σ^m by Bayes rule

Example



Example

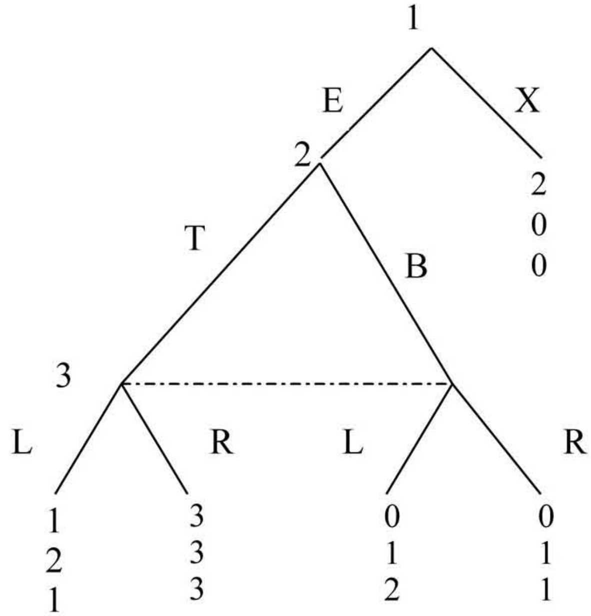


1
ε_1
$1-\varepsilon_1$
E
X
2
2
0
0
T
$1-\varepsilon_2$
B
ε_2
0.1
0.9
3
0
L
R
L
R
1
2
1
3
3
3
0
1
2
0
1
1

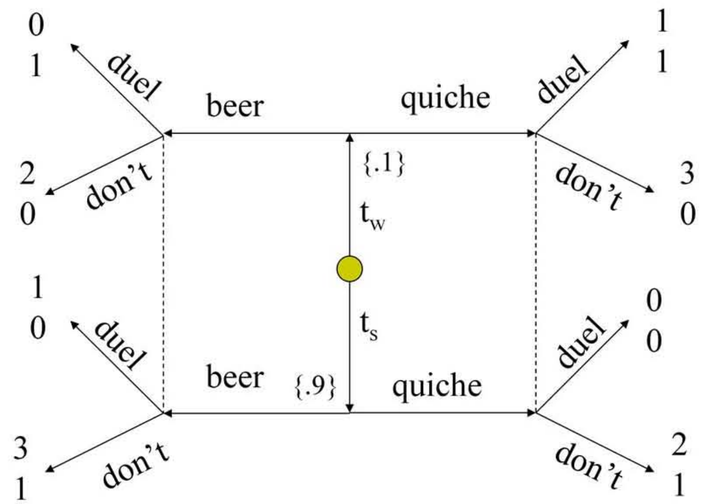
Sequential Equilibrium

- An assessment: (σ,μ) where σ is a strategy profile and μ is a belief system, μ(*h*)∈Δ(h) for each *h*.
- An assessment (σ,μ) is sequentially rational if at each h_i, σ_i is a best reply to σ_{-i} given μ(*h*).
- (σ,μ) is consistent if there is a sequence (σ^m,μ^m) →(σ,μ) where σ^m is "completely mixed" and μ^m is computed from σ^m by Bayes rule
- An assessment (σ,μ) is a sequential equilibrium if it is sequentially rational and consistent.

Example

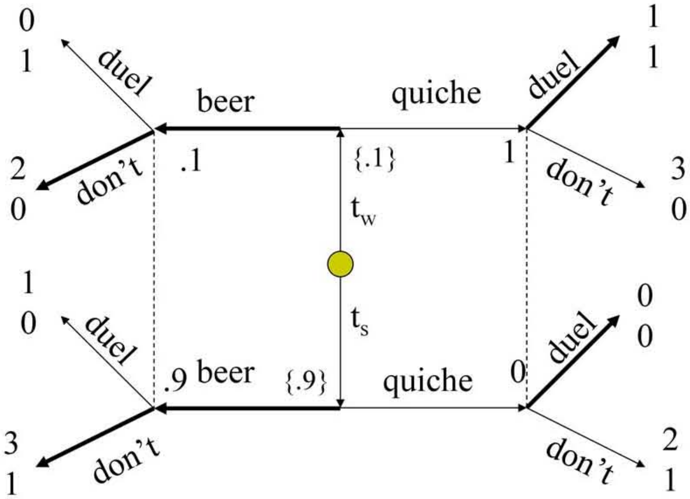


Beer – Quiche

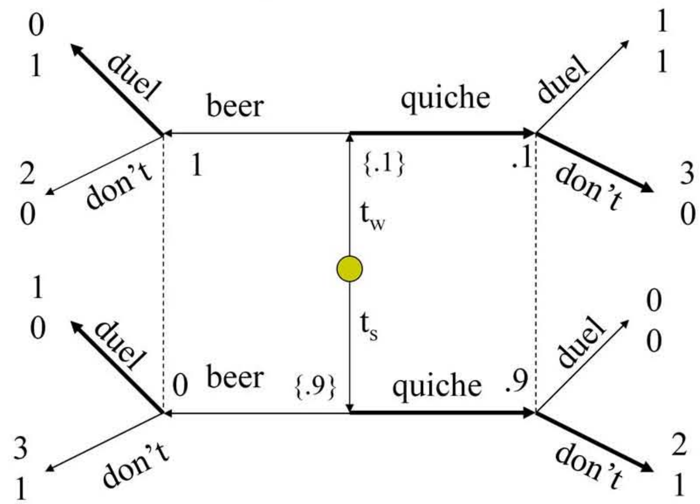


0
1
duel
beer
quiche
duel
1
1
{.1}
2
0
don't
don't
3
0
t_w
1
0
duel
t_s
duel
0
0
beer
{.9}
quiche
3
1
don't
don't
2
1

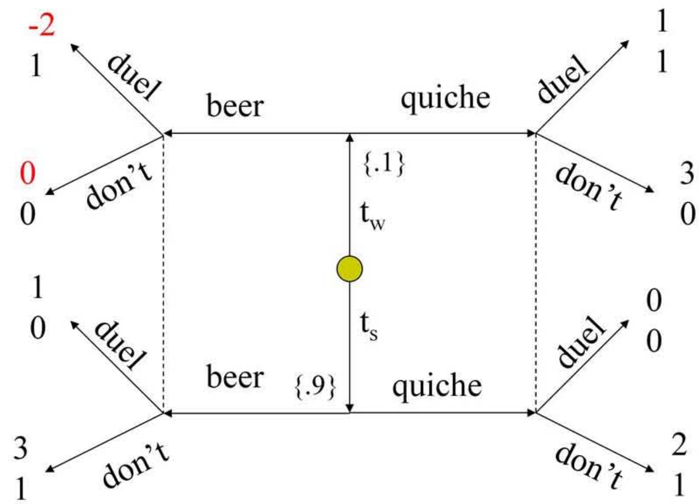
Beer – Quiche, An equilibrium



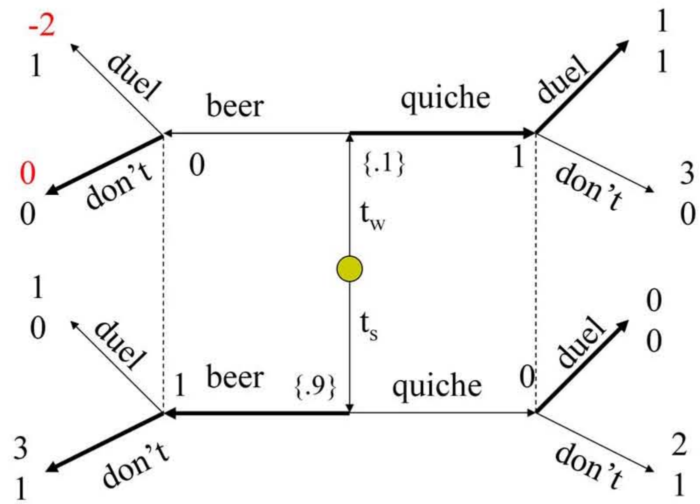
Beer – Quiche, Another equilibrium



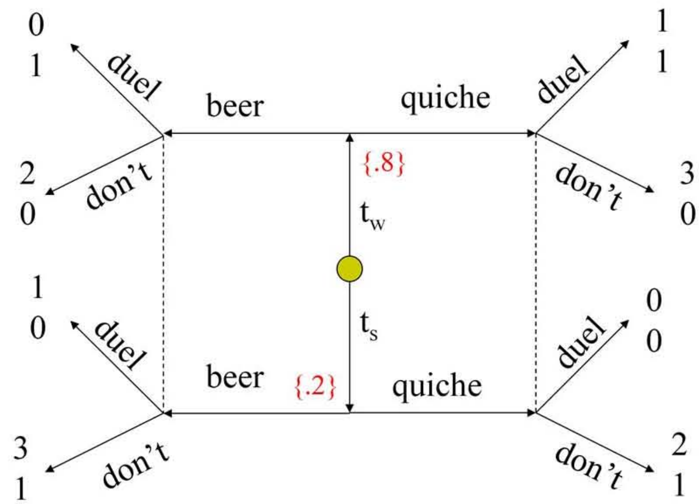
Beer – Quiche, revised



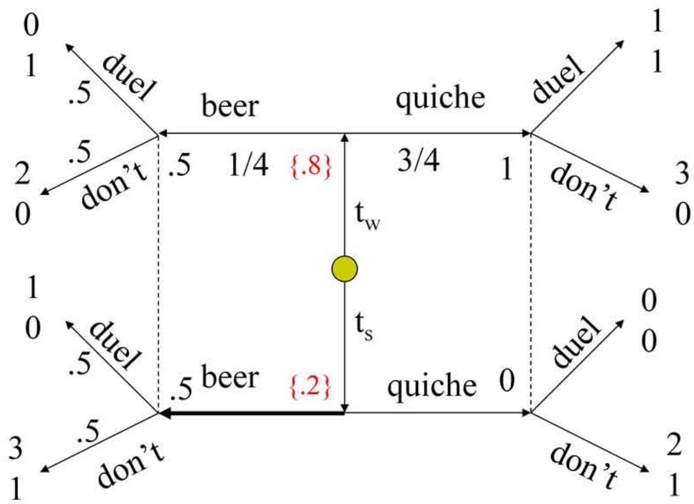
Beer – Quiche, Revised



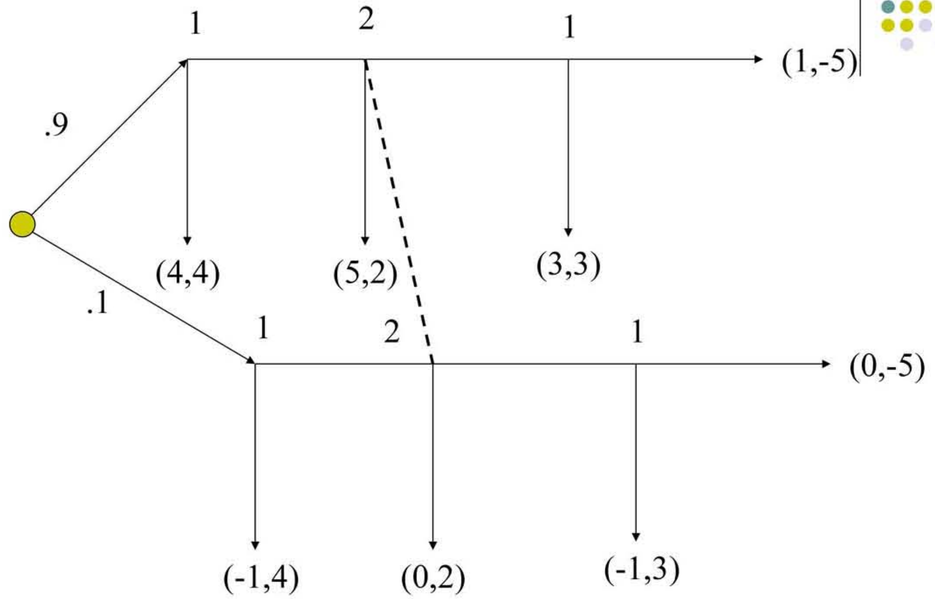
Beer – Quiche in weakland



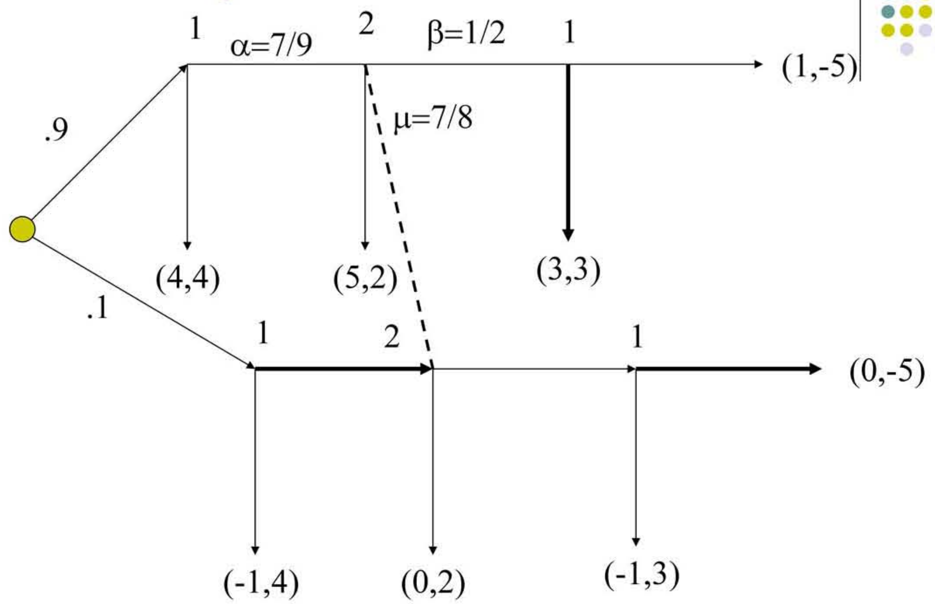
Beer – Quiche in weakland
Unique PBE



Example



Example – solved

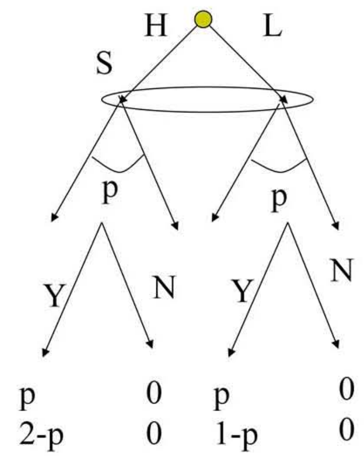


Sequential Bargaining

1. 1-period bargaining – 2 types
2. 2-period bargaining – 2 types
3. 1-period bargaining – continuum
4. 2-period bargaining – continuum

Sequential bargaining 1-p

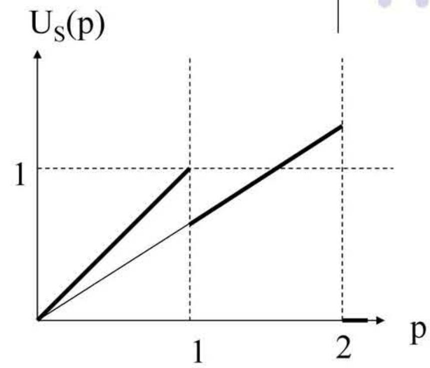
- A seller S with valuation 0
- A buyer B with valuation v;
  - B knows v, S does not
  - v = 2 with probability π
  - = 1 with probability $1-\pi$
- S sets a price $p \geq 0$;
- B either
  - buys, yielding (p,v-p)
  - or does not, yielding (0,0).



Solution

1. B buys iff $v \geq p$;
   1. If $p \leq 1$, both types buy: S gets p.
   2. If $1 < p \leq 2$, only H-type buys: S gets πp.
   3. If $p > 2$, no one buys.
2. S offers

- 1 if $\pi < ½$,
- 2 if $\pi > ½$.



Sequential bargaining 2-period

- A seller S with valuation 0
- A buyer B with valuation v;
  - B knows v, S does not
  - v = 2 with probability π
  - = 1 with probability $1-\pi$

1. At t = 0, S sets a price $p_0 \geq 0$;
2. B either
   - buys, yielding $(p_0, v-p_0)$
   - or does not, then
3. At t = 1, S sets another price $p_1 \geq 0$;
4. B either
   - buys, yielding $(\delta p_1, \delta(v-p_1))$
   - or does not, yielding (0,0)

Solution, 2-period

1. Let μ = Pr(v = 2|history at t=1).
2. At t = 1, buy iff $v \geq p$;
3. If $\mu > ½$, $p_1 = 2$
4. If $\mu < ½$, $p_1 = 1$.
5. If $\mu = ½$, mix between 1 and 2.
6. B with v=1 buys at t=0 if $p_0 \leq 1$.
7. If $p_0 > 1$, $\mu = Pr(v = 2|p_0,t=1) \leq \pi$.

Solution, cont. π <1/2

1. $\mu = \Pr(v = 2 | p_0, t=1) \leq \pi < 1/2$.
2. At t = 1, buy iff $v \geq p$;
3. $p_1 = 1$.
4. B with v=2 buys at t=0 if

   $$(2-p_0) \geq \delta(2-1) = \delta \Leftrightarrow p_0 \leq 2-\delta.$$

5. $p_0 = 1$:

   $$\pi(2-\delta) + (1-\pi)\delta = 2\pi(1-\delta) + \delta < 1-\delta+\delta = 1.$$

Solution, cont. $\pi > 1/2$

- If v=2 is buying at $p_0 > 2-\delta$, then
  - $\mu = Pr(v = 2|p_0 > 2-\delta, t=1) = 0$;
  - $p_1 = 1$;
  - v = 2 should not buy at $p_0 > 2-\delta$.
- If v=2 is not buying at $2 > p_0 > 2-\delta$, then
  - $\mu = Pr(v = 2|p_0 > 2-\delta, t=1) = \pi > 1/2$;
  - $p_1 = 2$;
  - v = 2 should buy at $2 > p_0 > 2-\delta$.
- No pure-strategy equilibrium.

Mixed-strategy equilibrium, $\pi > 1/2$

1. For $p_0 > 2-\delta$, $\mu(p_0) = ½$;
2. $\beta(p_0) = 1 - \Pr(v=2 \text{ buys at } p_0)$

$$\mu = \frac{\beta(p_0)\pi}{\beta(p_0)\pi + (1-\pi)} = \frac{1}{2} \Leftrightarrow \beta(p_0)\pi = 1-\pi \Leftrightarrow \beta(p_0) = \frac{1-\pi}{\pi}.$$

3. v = 2 is indifferent towards buying at p_0:

   $2 - p_0 = \delta\gamma(p_0) \Leftrightarrow \gamma(p_0) = (2 - p_0)/\delta$

   where $\gamma(p_0) = \Pr(p_1=1|p_0)$.

Sequential bargaining 1-period

- A seller S with valuation 0
- A buyer B with valuation v;
  - B knows v, S does not
  - v is uniformly distributed on [0,1]
- S sets a price $p \geq 0$;
- B either
  - buys, yielding (p,v-p)
  - or does not, yielding (0,0).

Sequential bargaining, v in [0,a]

- 1 period:
  - B buys at p iff $v \geq p$;
  - S gets $U(p) = p \Pr(v \geq p)$;
  - v in [0,a] => $U(p) = p(a-p)/a$;
  - $p = a/2$.

Sequential bargaining 2-periods

If B does not buy at t = 0, then at t=1

- S sets a price $p_1 \geq 0$;
- B either
  - buys, yielding $(\delta p_1, \delta(v-p_1))$
  - or does not, yielding (0,0).

Sequential bargaining, v in [0,1]

- 2 periods: (p_0, p_1)
  - At t = 0, B buys at p_0 iff $v \geq a(p_0)$;
  - $p_1 = a(p_0)/2$;
  - Type $a(p_0)$ is indifferent:

$$a(p_0) - p_0 = \delta(a(p_0) - p_1) = \delta a(p_0)/2$$

$$\Leftrightarrow a(p_0) = p_0/(1-\delta/2)$$

  - S gets

$$\left(1-\frac{p_0}{1-\delta/2}\right)p_0 + \delta\left(\frac{p_0}{2-\delta}\right)^2$$

  - FOC:

$$1-\frac{2p_0}{1-\delta/2}+\frac{2\delta p_0}{2-\delta}=0 \Rightarrow p_0=\frac{(1-\delta/2)^2}{2(1-3\delta/4)}$$

14.12 Economic Applications of Game Theory
Fall 2012