14.121 Problem Set #1

Due September 14, 2005

1. Let P be a preference relation on a set X. Assume that P is complete, reflexive and transitive. Let the binary relation $\succcurlyeq_P$ represent P. Define two new binary relations on X, denoted $\succ_P$, by

$$\begin{aligned} x \sim_P y &\Leftrightarrow (x \succcurlyeq_P y) \wedge (y \succcurlyeq_P x); \\ x \succ_P y &\Leftrightarrow (x \succcurlyeq_P y) \wedge \neg (y \succcurlyeq_P x). \end{aligned}$$

Prove the following:

1. $\sim_P$ is symmetric: If $x \sim_P y$ then $y \sim_P x$.
2. $\sim_P$ is reflexive: $x \sim_P x$.
3. $\sim_P$ is transitive: If $x \sim_P y$ and $y \sim_P z$ then $x \sim_P z$. Together, the first three parts have shown that $\sim_P$ is an equivalence relation.
4. Show that if $x \succ_P y$ and $y \succcurlyeq_P z$ then $x \succ_P z$.

2. (MWG Exercise 1.B.3) Show that if $f : \Re \to \Re$ is a strictly increasing function and $u : X \to \Re$ is a utility function representing preference relation $\succcurlyeq_P$, then the function $v : X \to \Re$ defined by $v(x) = f(u(x))$ (that is, $v = f \circ u$) is also a utility function representing preference relation $\succcurlyeq_P$.

3. Let $\succcurlyeq_P$ be a complete, reflexive and transitive preference relation on X. We showed in class that if X is finite, then there is a utility function u that represents $\succcurlyeq_P$. In this question we'll see what happens when X is (uncountably) infinite.

Suppose X is $\Re^2$ and $\succcurlyeq_P$ is defined by $(x_1, x_2) \succcurlyeq_P (y_1, y_2)$ iff $x_1 > y_1$ or ($x_1 = y_1$ and $x_2 \geq y_2$). These preferences are called *lexicographic*.

(a) Show that $\succcurlyeq_P$ defines a complete, reflexive and transitive preference relation on X.

(b) Show that there is no utility function $u : X \to \Re$ that represents $\succcurlyeq_P$. What does this mean for the theorem that we proved in class (that if X is finite and $\succcurlyeq_P$ is complete, reflexive and transitive, then there is a utility function that represents $\succcurlyeq_P$)?

(c) Theorem 1.1 in the Jehle-Reny book shows that if $X = \Re^n$ and $\succcurlyeq_P$ is a complete, reflexive, transitive preference relation on X satisfying two additional axioms (continuity and strict monotonicity), then $\succcurlyeq_P$ can be represented by a utility function.

Which of these properties are violated by lexicographic preferences?

4. The paper on the reading list by Carson, Wilks, and Imber attempts to place a dollar value on the preservation of the Kakadu Conservation Zone by surveying 2034 Australians about their preferences. They find that the benefits of the project greatly outweigh the costs.

A critique of this methodology (found in the Diamond-Hausman paper among other places) is that the survey responses do not accurately reflect people's true preferences. There is no reason to lie in response to a survey, but people might misrepresent their preferences for any of several reasons: they have had little time to think about the issue and don't know their preferences well; they may worry about what the interviewer would think about the true preferences; or they may derive utility from thinking that they are a virtuous person who would contribute to the public good if asked.

It's not necessary to read though the papers, but at least try to skim through them and then try to see if you can use the material from this class to help think about how we could assess whether the survey results seem like true preferences over the Kakadu. What axioms for preferences might be violated by the survey results? How might one design a better survey to look for such violations? What do you think we should do if we find violations?