

Question 1

(a) See the lecture notes or FT for the formal definition. We care about continuity at infinity because if this property is satisfied, then checking a proposed SPE is equivalent to checking the single-deviation-property (ie that no player can achieve a higher payoff by deviating from their equilibrium strategy at a single node, and playing according to their equilibrium strategy at all other nodes).

(b) Yes, it has a unique NE (u, A) . This is also the only outcome that survives iterated strict dominance. To see this, start by noting that for player 2, C is strictly dominated by $\frac{3}{5}A + \frac{2}{5}B$ (or other combinations of A and B will work as well), and go from there.

(c) See the notes for the definitions. The assumption that breaks down in the “name the largest number” game is that Σ , the space of strategy profiles, is not compact (because there is no limit on how large the numbers named by the players can be).

(d) This is false. If a property holds for generic games, then for any game for which the property doesn't hold, there is another game, where the payoffs are arbitrarily close to the first game, for which the property does hold. So clearly this isn't true in, for example, games where all the payoffs are strictly negative.

(e) There are two subgames (three if you also count the whole game as a subgame). One begins at 1's node just above the terminal nodes (1,1) and (2,0). The other begins at the upper of 2's non-singleton nodes.

(BTW: According to the way FT, and I believe Glenn, defines things, all games are proper subgames of themselves, so by this definition the whole game is also a subgame. Gibbons, on the other hand, doesn't count the original game as being one of the subgames).

(f) The centipede game that Glenn discussed in class is an example where SPE/backward induction rules out a reasonable outcome. If you remember, the SPE is to play down immediately, giving the players a low payoff. But it is probably more reasonable from an empirical point of view to expect players to cooperate for a while before they play down.

An example where SPE gives an unreasonable result - Glenn gives several of these in the notes. He discusses games in which SPE allows equilibria that involve players choosing actions at an information set that are inconsistent with any possible set of beliefs the player could hold at that information set. Signalling games will often have unreasonable SPE, because there are no subgames (you can't separate parts of the game tree without cutting information sets).

(g) Firstly note that D and then B can be eliminated by iterated strict dominance. There are no pure strategy NE of the resulting 2×2 game. The mixed strategy NE is $(\frac{1}{3}U + \frac{2}{3}M, \frac{2}{5}A + \frac{3}{5}C)$.

(h) The NE is $a_1 = \frac{32}{15}, a_2 = \frac{38}{15}$ (found by solving the two best response functions simultaneously).

(i) Remember for the Intuitive Criterion you basically have to establish two things - heuristically (1) that the type making the speech gets a higher payoff under the deviation and some best response to the deviation than under the PBE, and (2) that the other type does not have an incentive to make such a speech. So Irving's argument is ok - you can verify that the $\theta = 3$ type could do better by choosing $e = 1$, but the $\theta = 2$ type could not. Freddy's argument doesn't make sense, because neither type has an incentive to make such a speech.

Question 2

- (a) Tree not drawn. But player 1 has $3 \times 2 \times 2 \times 2 = 24$ pure strategies, and player 2 has $3 \times 3 \times 3 = 27$ pure strategies.
- (b) For example, can check that the strategy profile where player 1 plays: (*give other player \$2, D, D, D*), and player 2 plays: (*B if player 1 gave up 2, C otherwise*) is a NE. Given the strategy of the other player, neither player can achieve a higher payoff by unilaterally adopting a different strategy.
- (c) If $x < 0$ then (U, A) is not a NE of the subgame, and thus the only NE of the subgame is (D, B) . Then by backward induction player 1 will choose to give up 0 at the first stage of the game. So the condition for a unique SPE is $x < 0$.
- (d) We are looking for an SPE where the static NE (U, A) will be used to reward player 1 when he gives up \$2 at the first stage of the game (we can't use (U, A) as the punishment NE because $x \geq 0$ so the punishment won't be big enough). So along the equilibrium path, player 1 will get payoff of x , which must be greater than $2 + 1$ (since the second stage outcome must always be a NE of the second stage game).
So $x \geq 3$ will ensure that the strategy profile where player 1 plays (*give other player \$2, A if 1 gave away \$2 at the first stage, D otherwise*) and player 2 plays (*A if player 1 gave up \$2, B otherwise*) is an SPE.
- (e) No, because the lowest-payoff NE in the second stage gives payoff of at least zero to both players (zero occurs only when $x = 0$). So player 1 can always keep both dollars at the first stage of the game, and in any SPE will at worst get 0 in the second stage, for a total payoff of 2.

Question 3

- (a) $\Theta_1 = \{\text{type who doesn't know } q\}$. $\Theta_2 = \{1, 2, 3\}$. MIT has $2 \times 2 \times 2 = 8$ pure strategies (a choice between two actions for each of three types), while Harvard has only 2 pure strategies.
- (b) The actions $a_2(q = 3) = \text{don't admit}$, and $a_2(q = 1) = \text{admit}$ are both conditionally dominated. In words, MIT will never let in a $q = 1$ student because even if Harvard does not make the student an offer, the payoff for MIT is less than 0. In contrast, MIT will always admit a $q = 3$ student regardless of Harvard's strategy, because even if Harvard makes the student an offer, MIT's payoff is $0.65 \times 1.5 - 1 \times 0.35$, which is greater than zero. No actions are conditionally dominated for MIT when $q = 2$, because they want to admit the student if and only if Harvard does not admit the student.
Harvard has no conditionally dominated strategies.
- (c) Given the discussion in part (b), the only possible BNE's are
1. $a_1 = \text{Admit}; a_2(q = 1) = a_2(q = 2) = \text{Don't admit}, a_2(q = 3) = \text{Admit}$
 2. $a_1 = \text{Don't admit}; a_2(q = 1) = \text{Don't admit}, a_2(q = 2) = a_2(q = 3) = \text{Admit}$

You can check that the first of these possible equilibria doesn't work. In the first putative equilibrium, Harvard's expected payoff from admitting the student is

$U(\text{Admit}) = -0.5 \times 1/3 + 0.5 \times 1/3 + (0.35 \times 1.5 - 1 \times 0.65) \times 1/3$, which is negative. So Harvard would choose to deviate and not admit the student.

By similar calculations, can check that the second potential equilibrium really is a BNE.

So the unique BNE will be $a_1 = \text{Don't admit}; a_2(q = 1) = \text{Don't admit}, a_2(q = 2) = a_2(q = 3) = \text{Admit}$.

- (d) No, observing MIT's action won't help. If $q = 2$ or $q = 3$ (ie if Harvard observes ADMIT), Harvard will make a negative expected payoff trying to compete for the student. If $q = 1$ (Harvard observes NO ADMIT), Harvard is better off not admitting the student.

Question 4

(a) **No**, because in this case $p_{HMO} = 500$ and $p_{FULL} = 2000$. So the unhealthy type gets utility 0 from choosing full insurance, and utility 500 from choosing the HMO. Consequently, they would deviate from their equilibrium strategy.

(b) The pooling equilibrium we're looking for is:

$$\begin{aligned} s_1(\theta) &= FULL \text{ for } \theta \in \{healthy, not\ healthy\} \\ s_2(FULL) &= 1000q + 2000(1 - q) \\ s_2(HMO) &= 1000 \\ \mu(\theta = healthy \mid s_1 = FULL) &= q \\ \mu(\theta = healthy \mid s_1 = HMO) &= 0 \end{aligned}$$

Note that player 2's beliefs are the same as her priors, because this is a pooling equilibrium. Off the equilibrium path, we want player 2 to have the 'worst' possible beliefs, so that the set of admissible q 's is as large as possible. Also note that the price for full insurance is equal to the expected payout, because of the quadratic nature of player 2's utility function.

Now have to check this is an equilibrium. Firstly, player 2 will not deviate, since they are getting their maximum possible payoff.

Secondly, the unhealthy type has no incentive to deviate from this equilibrium, since the price for full insurance is less than 2000 (if they chose the HMO option, their utility would be zero).

What about the healthy type? They will not deviate as long as:

$$2000 - [1000q + 2000(1 - q)] \geq 1500 - 1000$$

This expression simplifies to $q \geq 0.5$

(c) In this case you might expect the equilibrium to take the form of a cutoff rule - player 1 will choose full insurance when their cost $c > c^*$.

For the cutoff type: $2000 - c^*/2 = p_{HMO} = 2000 - p_{FULL}$. Also note that the prices for both types of health insurance must reflect the expected payouts, so:

$$\begin{aligned} p_{FULL} &= \frac{1}{2}(c^* + 2000) \\ p_{HMO} &= \frac{1}{2}c^* \end{aligned}$$

Substituting these prices into our cutoff rule, we find that $c^* = 2000$, which implies that with probability 1, full insurance is not taken up.

(d) The model shows that because of adverse selection, private insurance markets are often unable to provide a high enough level of insurance. To think about the problem more seriously, would need to take into account the possibility of screening by health insurers, as well as the risk aversion of individuals (which will determine the optimal demand for insurance - in this model there is no real need for insurance in the first place because everyone is risk-neutral).

