

HOMEWORK 4

1. Consider the following regulation problem. Let x be output, c be marginal cost of output and R be net return of production. The firm's net profit before regulatory payments is given by

$$R = x - cx, \text{ for } 0 \leq x \leq 1.$$

Note that x has to lie in the unit interval and that output has been normalized to equal revenue. Assume that the regulator does not know the firm's cost coefficient c , which can take on n values: $0 \leq c_1 \leq \dots \leq c_n \leq 1$. The regulator assesses probability $p_i \geq 0$ to each of the i possible marginal cost levels. The firm knows its marginal cost and chooses x . The regulator can tax the firm based on its realized revenue.

- a. Set up the program that solves for the regulator's optimal policy, expressed as a direct revelation mechanism. Assume that the regulator's objective is to extract as much surplus from the firm as possible, subject to the constraint that the firm can quit if profits are negative (and pay nothing).
- b. Draw a diagram that shows how the firm chooses its best response as a function of its cost. Prove that the firm's output choice x is a non-decreasing function of marginal cost.
- c. Show that the optimal policy takes the simple form: a fixed charge T for producing anything (hence, all firms with $c \leq T$ produce and those with $c > T$ do not). It suffices that you give an argument for $n = 3$.

*2. Consider a screening model of the following sort. The agent produces output q at a cost $c(q, \beta) = q\beta$, where q is output and β is a cost parameter. The principal, who cannot observe either c or β , offers to pay the agent the amount $p(q)$ if the agent produces output q . Given this incentive scheme, an agent with cost parameter β responds by producing the amount $q(\beta)$.

What kind of payment scheme $p(q)$ should the principal choose in order to implement the response function $q(\beta) = 1/\beta^2$?

*3A seller may supply a single object to a buyer. Let x be 1 if trade takes place and 0 if not. Let t_s be the amount of money that the seller *receives* and t_b the amount that the buyer *pays*. Let v be the buyer's valuation and s be the seller's valuation of the object and assume preferences are quasi-linear. We can then normalize utilities so that the seller's utility is $t_s - cx$ and the buyer's utility is $vx - t_b$. Assume the preference parameters v and c are independently drawn from a uniform distribution on $[0,1]$. The buyer knows v and the seller knows c .

- a. What is the efficient rule for trade?
- b. Let m_s and m_b be the seller's respectively the buyer's reported preference for the object. Determine the set of direct mechanisms (expressed as a function of the reports) that admit truth telling as a dominant strategy and implement efficient trade (ie. the set of Groves mechanisms).
- c. Show that there is a unique Groves mechanism that has the property that whenever trade does not occur, the transfer payments are set equal to zero ($t_b = t_s = 0$). Is this mechanism feasible?
- d. Show that there is no Groves mechanism for which the budget breaks even for all reported preferences.

4. Suppose there are a continuum of sellers and a continuum of buyers (where each continuum is normalized to one). Each seller has one unit of the good and has valuation v_s drawn (independent from those of the other sellers and buyers) from the distribution F_s on $[a,b]$; similarly, each buyer has unit demand and has valuation v_b drawn (independently) from the distribution F_b on $[c,d]$. Assume that $b > c$ (not everyone ought to trade). Let $m_s(v_s)$ and $m_b(v_b)$ denote a seller's probability of selling and a buyer's probability of buying, respectively. Let $t_s(v_s)$ and $t_b(v_b)$ denote a seller's transfer and a buyer's transfer (from the planner), respectively. Among the many possible mechanisms that might be used, consider the "Walrasian mechanism":

$$m_s(v_s) = 1 \text{ and } t_s(v_s) = p \text{ if } v_s \leq p \text{ and } m_s(v_s) = 0 = t_s(v_s) \text{ if } v_s > p,$$

$$m_b(v_b) = 1 \text{ and } t_b(v_b) = -p \text{ if } v_b \leq p \text{ and } m_b(v_b) = 0 = t_b(v_b) \text{ if } v_b > p.$$

- a. Show that IC and IR are satisfied for any p .
- b. Show that there is a value of p such that the mechanism is balanced and trade is efficient.
- c. The Myerson-Satterthwaite Theorem states that it is impossible to achieve efficient and individually rational trading in a Bayesian NE between a buyer and a seller with private information about their valuations. How can this be reconciled with parts a and b above?

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