Problem Set 4

Problem 1: Let $(N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N})$ be a normal form game. Suppose for each i, the set of actions S_i is a compact set, and each u_i is continuous. Let $S_i^0 = S_i$, and define

$$S_i^k = \{s_i \in S_i^{k-1} \mid \nexists s_i' \in S_i^{k-1} \text{ that dominates } s_i\}.$$

Show that $S_i^\infty = \cap_{k=0}^\infty S_i^k$ is non-empty for each i.

Problem 2: Let $(N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N})$ be a normal form game. Suppose for each i that S_i is a compact and convex subset of a Euclidean space, that $u_i(s_i, s_{-i})$ is continuous in s_{-i}, and that $u_i(s_i, s_{-i})$ is continuous and concave in s_i. Use Kakutani's theorem to show that a pure strategy Nash Equilibrium exists.

Problem 3 (Cournot Competition with Different Costs): Suppose each of two firms produces a homogeneous good and the two firms simultaneously choose quatities $q_1, q_2 \in (0, \infty)$ to produce. Inverse demand given total quantity $Q = q_1 + q_2$ is $P(Q) = a - Q$ for some $a > 0$. Both firms have constant marginal costs of production, but firm 1 has a higher cost: $c_1 > c_2$.

(a) Find the Nash equilibrium of this game.

(b) Which firm produces more output in equilibrium?

(c) How do the outputs of the two firms change if we lower c_2?

Problem 4 (Bertrand Competition with Discrete Pricing): Suppose each of two firms produces a homogeneous good at constant marginal cost $\frac{1}{2}$. The two firms simultaneously set integer valued prices (that is, $p_i \in \{0, 1, 2, 3, ...\}$ for each firm $i \in \{1, 2\}$). Total demand is 1 at any price less than 4, and total demand is zero if the price is 4 or higher.
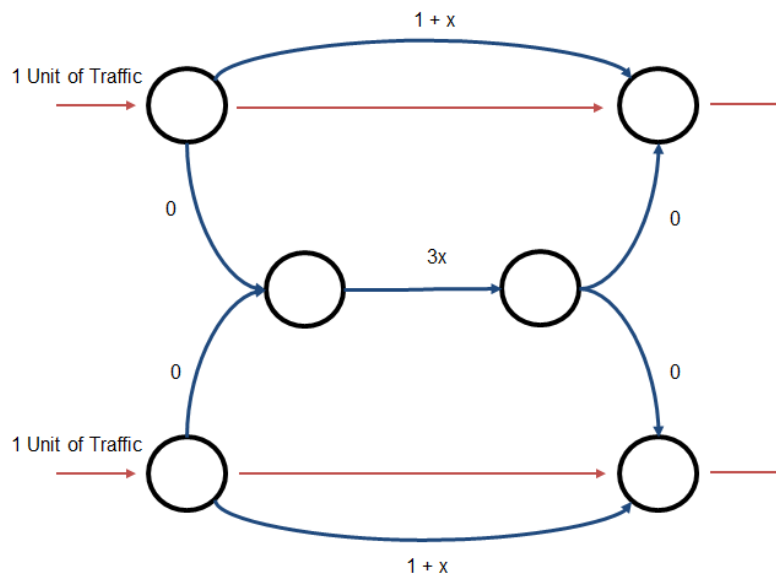
(a) Suppose the two firms split demand evenly if they choose the same price; otherwise the full demand goes to the lower priced firm. Charactcerize the set of pure-strategy Nash equilibria.

(b) Suppose firm 1 is the "incumbent," and will retain all demand unless firm 2 undercuts firm 1's price. That is, if the two firms charge the same price, firm 1 receives the full demand. Characterize the set of pure-strategy Nash equilibria.

(c) Does firm 1 benefit from its status as the "incumbent?"

Problem 5 (A Graphical Coordination Game): Let N denote a finite set of players. The players are linked in an undirected graph G in which some edges are colored red and other edges are colored blue. For player i, let $N_{i,r}$ denote the set of players linked to i along red edges, and let $N_{i,b}$ denote the set of players linked to i along blue edges. Players simultaneously choose one of two actions, A or B, so $S_i = \{A, B\}$ for each i. Player i's payoff is

$$u(s_i) = |\{j \in N_{i,r} \mid s_j = s_i\}| - |\{j \in N_{i,b} \mid s_j = s_i\}|.$$

That is, player i earns a unit of utility for each red neighbor she matches and loses a unit of utility for each blue neighbor she matches. Show that this is an (exact) potential game.

Problem 6: Consider the traffic flow game pictured in the figure below. There are two origin-destination pairs. A unit of traffic needs to flow from the upper left to the upper right, and another unit needs to flow from the lower left to the lower right. The cost functions are given in the figure.

1 + x

1 Unit of Traffic

0

0

3x

0

0

1 Unit of Traffic

1 + x

(a) What is the socially optimal routing, and what is its total cost?

(b) What is the equilibrium routing? What is the welfare loss relative to the optimum?

(c) Suppose you can impose constant tolls on some edges and constant subsidies on others. Design a system of tolls and subsidies to implement the social optimum as an equilibrium.

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