

1. The town of Economia has two residents: Ann and Bob. The town currently funds its fire department solely from the individual contributions of these residents. Each of the two residents has a utility function over private goods ( $X$ ) and total firemen ( $M$ ), of the form:  $U = 2 \log(X) + \log(M)$ . The total provision of firemen hired,  $M$ , is the sum of the number hired by each of the two persons:  $M = M_A + M_B$ . Ann and Bob both have income of 200, and the price of both the private good and a fireman is 1. They are limited to providing between 0 and 200 firemen. For the purposes of this problem, you can treat the number of firemen as a continuous variable (it could be man-years).

- (a) How many firemen are hired if the government does not intervene? How many are paid for by Ann? By Bob?
- (b) What is the socially optimal number of firemen? If your answer differs from a), explain why.
- (c) Suppose the government is not happy with the private equilibrium, and it decides to provide 20 more firemen. It taxes Ann and Bob equally to pay for the new hires. What is the new total number of firemen? How does your answer compare to part a? Have we achieved the social optimum? Why or why not?
- (d) Suppose now that the government is still not happy, so it decides to provide 75 new firemen. It taxes Ann 50 to pay for them, and it taxes Bob 25. What is the new total number of firemen? How many are provided by Ann, and how many by Bob? How does this compare to the level of provision in c), and why?

2. A profit-maximizing company has the chance to research the cure for the common cold (CCC). If it spends  $Z > 1$  on R&D, then it has probability  $p(Z) = 1 - 1/Z$  of discovering the CCC (and zero probability otherwise). Assume that the company has no other costs besides research costs, no other revenues other than those from selling the CCC if it discovers it. The present value (PV) of revenue from the CCC once discovered is  $II > 5$ . Units for research costs, and CCC profits are millions of dollars.

- (a) How much will the company spend on research as a function of  $II$ ?
- (b) What are the expected profits in terms of  $II$ ? What happens if  $II < 4$ ?
- (c) Assuming no discounting, so that the PV of profits is just the sum of all future profits, and an annual profit of \$0.15 million from monopoly exploitation of CCC, what is the minimum patent length that will induce the company to do any research?
- (d) Assume the patent is the length you worked out in the previous section. What is the maximum license fee the company could charge for the CCC?

3. Assume that the market demand for watches is  $Q=160-P$  and that the constant average cost of production is 80. Consider two alternatives. In case C, the industry is initially organized competitively, and in case M, the industry is organized as a monopoly. In each case, an invention leads to a lower, constant average cost of production of 60. (The R&D cost of the invention is not relevant to this problem, inasmuch as the issue is the magnitude of the incentive to invent in the two cases). Finally, there is no rivalry to make the invention: in case C, the inventor may be assumed to have a patent of infinite life, and in case M, the inventor is the existing monopolist, and entry is barred.

- (a) Find the initial equilibrium price and quantity in cases C and M.
- (b) What is the return that the inventor in case C could expect from a lower cost production process? Assume that the inventor monopolizes the watch business.
- (c) What is the return to the monopolist in case M that results from inventing the lower cost production process?
- (d) Interpret your results regarding the incentive to invent under monopoly and competition.
- (e) In case C, if society could require that the new process be used efficiently – by making it freely available to all – what would be the increase in total economic surplus? How does this magnitude compare to the returns found in b and c?
- (f) Consider the same situation as above, but where the invention lowers average cost to 5 rather than 60. Does this change the comparative incentives under monopoly and competition?