

1. Public good provision.

- (a) Each resident $i \in \{Ann, Bob\}$ maximizes utility subject to her or his budget constraint, taking as given the amount of firemen provided by the other resident (denoted by j).

$$\text{maximize } \{2 \ln(X_i) + \ln(M_i + M_j)\} \text{ subject to } \{X_i + M_i \leq 200\}. \quad (1)$$

The residents want to spend all of their money (it has no other use) so the budget constraint can be solved for $M_i = 200 - X_i$, leaving us with a maximization objective with just one variable:

$$2 \ln(X_i) + \ln(200 - X_i + M_j). \quad (2)$$

The FOC is

$$\frac{2}{X_i} - \frac{1}{200 - X_i + M_j} = 0. \quad (3)$$

Solving for X_i yields

$$X_i = \frac{400}{3} + \frac{2}{3}M_j. \quad (4)$$

Plugging this into the budget constraint gives $M_i = \frac{200}{3} - \frac{2}{3}M_j$.

Given the symmetry of the problem, we know that $M_i = M_j$ in equilibrium, so $M_i = 40$, and the total number of firemen provided is $M^* = (M_i + M_j) = 80$. Residents use the rest of their budget to consume $X_i = X_j = 120$.

- (b) To find the socially optimum number of firemen, we maximize the entire surplus of *Economia*, which is simply the sum of the two residents' utilities subject to the budget constraint based on the income of both residents. Given the symmetry of the problem we can set $X_i = X_j \equiv X$:

$$\text{maximize } \{4 \ln(X) + 2 \ln(M)\} \text{ subject to } \{2X + M \leq 400\}. \quad (5)$$

Substituting $M = 400 - 2X$ from the budget constraint leaves a maximization objective with one variable

$$2 \ln(X) + \ln(400 - 2X). \quad (6)$$

The FOC is

$$\frac{2}{X} - \frac{2}{400 - 2X} = 0. \quad (7)$$

Solving for X yields

$$X = \frac{400}{3} = 133\frac{1}{3}. \quad (8)$$

Plugging this back to the budget constraint gives $M^{**} = 400 - 2\frac{400}{3} = 133\frac{1}{3}$.

Now, the free riding problem is solved, as the sum of the marginal rates of substitution (as opposed to the individual's marginal rate of substitution) is equated with the marginal rate of transformation. That is, the positive externality that each consumer has on the other is internalized.

- (c) The tax reduces the income of each resident by 10. Residents must also take into account that the government is now making sure that at least 20 firemen are provided. The individual maximization problem is now

$$\text{maximize } \{2 \ln(X_i) + \ln(M_i + M_j + 20)\} \text{ subject to } \{X_i + M_i \leq 190\}. \quad (9)$$

Substituting in M_i from the budget constraint, this becomes

$$2 \ln(X_i) + \ln(210 - X_i + M_j). \quad (10)$$

The FOC is

$$\frac{2}{X} - \frac{1}{210 - X_i + M_j} = 0. \quad (11)$$

Solving for X yields

$$X = \frac{420}{3} + \frac{2}{3}M_j. \quad (12)$$

Plugging this into the budget constraint gives $M_i = 50 - \frac{2}{3}M_j$.

Given the symmetry of the problem, we know that $M_i = M_j$ in equilibrium, so $M_i = 30$, and the total number of firemen provided is $M^* = (30 + 30 + 20) = 80$.

This is the same as the answer in part (a), which is below the social optimum, so intervention has not done anything to solve the free rider problem. The tax reduces the residents' income, but does not lead them to internalize externality.

- (d) Notice that the different tax for Ann and Bob means that the problem is no longer symmetric for the two residents.

Taking into account that the government provides 75 firemen with the tax income, the maximization problem of Bob is

$$\text{maximize } \{2 \ln(X_B) + \ln(M_A + M_B + 75)\} \text{ subject to } \{X_B + M_B \leq 175\}. \quad (13)$$

Substituting in M_B from the budget constraint, this becomes

$$2 \ln(X_B) + \ln(250 - X_B + M_A). \quad (14)$$

The FOC is

$$\frac{2}{X_B} - \frac{1}{250 - X_B + M_A} = 0. \quad (15)$$

Solving for X yields

$$X_B = \frac{500}{3} + \frac{2}{3}M_A. \quad (16)$$

Plugging this into the budget constraint gives $M_B = \frac{25}{3} - \frac{2}{3}M_A$.

The maximization problem of Ann is

$$\text{maximize } \{2 \ln(X_A) + \ln(M_A + M_B + 75)\} \text{ subject to } \{X_A + M_A \leq 150\}. \quad (17)$$

Substituting in M_A from the budget constraint, this becomes

$$2 \ln(X_A) + \ln(225 - X_A + M_B). \quad (18)$$

The FOC is

$$\frac{2}{X_A} - \frac{1}{225 - X_A + M_B} = 0. \quad (19)$$

Solving for X yields

$$X_A = 150 + \frac{2}{3}M_B. \quad (20)$$

Plugging this into the budget constraint gives $M_A = -\frac{2}{3}M_B$. But it is impossible to have a negative contribution, so $M_A = 0$, resulting in $M_B = \frac{25}{3} = 8\frac{1}{3}$ and total $M = 83\frac{1}{3}$.¹

The problem would have been mathematically the same if Bob were richer than Ann by 25 to begin with, and the government was taxing both equally. Then with the government providing 75 firemen only Bob would find it worthwhile to contribute voluntarily for additional firemen.

2. Innovation and patent length

- (a) The company will maximize its expected profits by choosing the level of R&D spending $Z \geq 0$. It gets a present value of profits Π with probability $p(Z)$. Expected present value of profits is therefore $V(Z) = p(Z)\Pi - Z$. Differentiation wrt Z leads to the first order condition:

$$p'(Z)\Pi - 1 = \frac{\Pi}{Z^2} - 1 = 0$$

The solution $Z^* = \sqrt{\Pi}$ is the optimal level of R&D spending.

- (b) At the optimal level of spending, Z^* , expected profits are:

$$\begin{aligned} V^* &= p(Z^*)\Pi - Z^* = (1 - 1/\sqrt{\Pi})\Pi - \sqrt{\Pi} \\ &= \sqrt{\Pi}(\sqrt{\Pi} - 2) \end{aligned}$$

This is increasing in Π for $\Pi > 1$. When $\Pi = 4$, $V^* = 0$. This means that if $\Pi < 4$, then the present value of doing research must be negative, and the company would spend nothing on researching the CCC.

- (c) After the patent expires anyone can produce the drug and there are no more profits. That means that for n years, the company earns 0.15 per year, where n is the patent length. The total PV of income from the CCC is $0.15n$. We saw in part b) that the profits must be at least 4 for , so the shortest patent length which lets the company break even solves $0.15n = 4 \implies n = 400/15 = 26\frac{2}{3}$.
- (d) Once it has discovered the CCC, the NPV of profits is $0.15n = 4$ (\$m). This is the maximum value of a license, since it is the maximum the firm buying the license could earn from selling the CCC.

¹The first posting of this problem included a typo, setting the number of firemen provided by the government at 70 instead of 75. This would mean that 5 fireman-years worth of tax money disappeared somewhere. Anyway, with that figure $M_B = 11\frac{2}{3}$ and $M = 81\frac{2}{3}$.

3. Innovation and market structure

- (a) In the competitive case price equals marginal cost, $P^c = c = 80$, and total quantity is $Q^C = 160 - 80 = 80$. No one makes a profit.
In the M case, the monopolist maximizes $Q(160 - Q - c)$ by choosing $Q^M = 80 - c/2 = 40$. The corresponding price is $P^M = 120$. The monopolist makes a profit of $40 \times (120 - 80) = 1600$.
- (b) The inventor has constant marginal cost of 60. The inventor cannot charge more than 80 or else it would be undercut by other firms, but monopolizes the whole market by setting the price at a whisker under 80. Quantity will be 80 as before, and the inventor will make a profit of $80 \times (80 - 60) = 1600$.²
- (c) The monopolist would now choose $Q^{*M} = 80 - 60/2 = 50$. The corresponding price is 110 and profits are $50 \times (110 - 60) = 2500$. However, the monopolist was already making a profit of 1600 before so the return from the innovation is only 900.
- (d) The monopolist has less incentive to innovate, because it had a monopoly already before the innovation.
- (e) If all firms had access to the new technology, then the price will fall all the way to the new marginal cost. There are no profits, so total economic surplus is just consumer surplus. This must be bigger than the returns in both b) and c), since there is no deadweight loss. (Of course if companies knew this beforehand there would not have been any incentive to invent a lower cost process in the first place, so this case is not very realistic).
- (f) If the cost falls to 5, then the desired monopoly quantity would be 77.5, and the monopoly price would be 82.5, which is still slightly more than the cost before the innovation. Profit would be $77.5 (82.5 - 5) = 6006$, which is 4406 more than without the innovation. In the competitive case the profit of the innovator would be $80 (80 - 5) = 6000$. A monopolist still has less incentives to innovate.

²Equality to the monopoly profit in part a) is a numerical coincidence and not a general result.