

Problem Set #5

Due November 6, 2013

1. Consider a model in which a mass m of consumers are uniformly distributed around a circle of radius 1. Some number of firms who will locate around the circle are able to produce a homogeneous good at a constant marginal cost of c . Each consumer receives utility $v - td - p$ if he purchases one unit of the good at a price of p from a firm located at a distance of d from his location, and has zero utility if he does not purchase the good.

(a) Let K^* be the number of firms that enter in the pure strategy Nash equilibrium I described in class. In addition to the pure strategy Nash equilibrium, the entry game also has a symmetric mixed strategy Nash equilibrium in which the firms randomize and enter with probability p^* . Show that the expected number of entrants in this equilibrium satisfies $E(Np^*) > K^* - 1$. Give intuition for why you'd guess that the expected number of entrants would be larger in the mixed strategy equilibrium and also discuss why this need not be completely correct.

(b) Work out an example to show that if firms choose whether or not to enter sequentially and if firms also commit to a location around the circle when they choose to enter then there may be pure strategy equilibria where fewer than K^* firms enter.

2. Consider the setup of the standard vertical differentiation model. Suppose that consumers have types θ distributed with unit density on $[1, 3]$ and that a consumer of type θ gets utility

$$u_i = \theta s_i - p_i$$

when he pays a price p_i and consumes a good of quality s_i . Suppose that firm 1 can produce a good of quality 2 at marginal cost of 0.

(a) Suppose firm 2 is a potential entrant. If firm 2 pays a fixed cost of K it will be able to produce a good of quality 1 at marginal cost 0 and the firms will then play a simultaneous move pricing game. What is firm 1's profit as a function of K ?

(b) Suppose now that there is a third potential entrant which could enter at cost K and produce a goods of quality $\frac{3}{2}$. Is there any positive value of K for which all three firms enter?

3. Try to read at least through section 4 of Andrew Sweeting's 2009 *RAND Journal of Economics* paper, "The Strategic Timing Incentives of Commercial Radio Stations: An Empirical Analysis Using Multiple Equilibria." Sweeting's paper can be thought of more generally as suggesting that the fact that a model has multiple equilibria can actually make it easier to estimate rather than harder. Some other IO applications where models have multiple equilibria are entry games (where we don't know if 1 will enter and 2 will stay out or vice versa) and one or two sided network externality models where we don't know if everyone will go to firm 1 or firm 2. Think about these or other situations that seem like they should have multiple equilibria and write a paragraph about whether Sweeting's approach seems likely or unlikely to be potentially useful.

4. Consider the following two period model of learning-by-doing. In each of two periods, the

demand for a nondurable good is given by $P(Q) = 4 - Q$ where Q is the total quantity of the good produced.

In the first period, firm 1 (a monopolist) produces quantity q_1^1 of a good at a constant marginal cost of 2. After the first period, firm 2 has the opportunity to pay a sunk cost of E and enter the market. If firm 2 enters, then in the second period firm 1 and firm 2 compete as Cournot duopolists (otherwise firm 1 is again a monopolist). Firm 2 has a constant marginal cost of 2. Because of the experience it gained in the first period, however, firm 1 can produce the good at a lower marginal cost. Write $MC(q_1^1)$ for the second period *marginal cost* of firm 1 when its first period output was q_1^1 and assume that $MC(q_1^1) \in [1, 2]$.

- (a) What are the firms' outputs and profits in the second period as a function of $MC(q_1^1)$.
 (b) Assume that the function relating first period output and second period marginal cost is

$$MC(q_1^1) = \begin{cases} 2 & \text{if } q_1^1 \leq 1 \\ 2\frac{1}{2} - \frac{1}{2}q_1 & \text{if } q_1^1 \in [1, 3] \\ 1 & \text{if } q_1^1 > 3. \end{cases}$$

Assume also that firm 2 observes q_1^1 before making its entry decisions and choosing its second period output. Show that if $E = \frac{16}{81}$ it is not optimal for firm 1 to choose a q_1^1 which is sufficiently large so as to deter entry.

(c) Again suppose $E = \frac{16}{81}$ (or any other value which is such that firm 1 wants to "accomodate" entry) and that firm 2 observes q_1^1 before choosing its second period output. What output level does firm 1 choose?

(d) Suppose now that firm 2 is unable to observe q_1^1 . Without doing the calculations, how would you expect firm 1's first period output to differ from the answer to part (c)? Would you expect it to be greater than or less than one? How would the answers to the qualitative parts of this question change if the firms engaged in price competition instead of Cournot competition?

5. Consider the following model of brand proliferation. A continuum of consumers (of mass 1) are located around a circle of circumference one. In the first period, firm 1 has the opportunity to introduce any number N of brands and position them anywhere it likes around the circle. The cost of doing this is NE_1 . Firm 2 then chooses whether to enter, in which case it introduces and positions a single brand at a cost of E_2 . If firm 2 enters, assume that there is differentiated product price competitions with consumers having value $v - td^2 - p$ for a product located at a distance d from them.

(a) If firm 1 introduces two brands at points which are opposite each other on the circle, and firm 2 introduces a single brand half way between two of these show that the equilibrium prices and profits are $p_1 = 7t/48$, $p_2 = 5t/48$, $\pi_1 = 49t/576 - 2E_1$, $\pi_2 = 25t/576 - E_2$. Explain intuitively why firm 1 chooses a higher price than firm 2.

(b) Find values of v , t , E_1 , and E_2 for which firm 1 would choose $N = 1$ if entry were not possible, but "overinvests" in brand proliferation and chooses $N = 2$ in this model to deter entry.

(c) Suppose we added a third stage to this game where firm 1 could withdraw any of its brands if it desired before price competition occurs (but not get back the sunk costs of introducing the

brands). Given the parameter values from part (b) show that if firm 2 were to introduce a brand located in exactly the same place as one of firm 1's brands, then firm 1 would in equilibrium withdraw that brand. What does this imply about the feasibility of entry deterrence through brand proliferation?

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