

Exam 1

14.30 Fall 2004

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Instructions: This exam is closed-book and closed-notes. You may use a simple calculator. Please read through the exam in order to ask clarifying questions and to allocate your time appropriately. You **must show** all your calculations. You have 90 minutes to complete the exam (90 points). Good Luck!

1. (23 points)

A. (15 points, 2.5 points each) True, False, or Uncertain. Please avoid long explanations (but, no explanation no points).

- i. A pair of two events, A and B , have to be disjoint or exhaustive.
- ii. If two events A and B are independent, then $P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$.
- iii. Let A and B be two events. Then, $P(A) = P(A \cap B^c) + P(A \cap B)$.
- iv. If the joint pdf of X, Y is $f(x, y) = ke^{x+y}$ for $0 < x < 1$ and $0 < y < 1$, then X and Y are independent.
- v. The pmf/pdf, $f_X(x)$, is the probability that the random variable X takes on the value x .
- vi. The only difference between $F(x)$ and $F(x|y)$ is that in the latter case you know the value of y , but the distribution of X is the same in both cases.

B. (8 points) Assume that the joint pdf of two random variables X and Y is given by: $f(x, y) = 8xy$ for $0 \leq x \leq y \leq 1$, and 0 otherwise. Explain how would you compute $P(X + Y < 1 \text{ and } Y < 0.5)$ and $P(X + Y < 1 \text{ and } Y < 0.6)$; you **do not** need to solve the integrals, just define them correctly (**do not** forget the limits of integration).

2. (22 points) You are the CEO of PlatesRUs license plate manufacturing conglomerate. State regulators tell you that each license plate must contain 2 letters (26 possible letters, A through Z) and 3 numbers (10 possible digits, 0 through 9).

a.(5 points) Suppose that each license plate must contain 2 letters **followed by** 3 numbers. Numbers and letters can repeat. How many different plates are possible? (Note: on a license plate AB123 is different from BA123.)

b.(7 points) Regulators have stated new rules: plates still contain 2 letters and 3 numbers, but now the numbers and letters can be in **any** of the 5 positions. However numbers and letters **do not** repeat on these new plates. (E.g., 1A23B is now valid, but 1A13B is not.) How many plates are possible now?

c.(10 points) Assume now that plates are still designed as in part **b**, except that the no-letter-or-number-repeat clause is not enforced any more. What is the probability that a given plate contains the numbers 777 in succession (i.e. right next to each other, with no letters in between)?

3. (23 points) The local barber shop is home to three barbers. The barbers appear identical but vary in their ability to provide good haircuts. Denote the event that you receive a good haircut by G , the event that you do not receive a good haircut by NG , and the event that your hair is cut by barber i by B_i , where $i \in \{1, 2, 3\}$. The probability that any particular barber cuts your hair is $\frac{1}{3}$. The probability that you receive a good haircut from barber 1 is 1. The probability that you receive a good haircut from barber 2 is $\frac{1}{2}$. The probability that you receive a good haircut from barber 3 is x , where $x \in [0, 1]$.

a.(5 points) Using the Bayes Theorem calculate the probability that your hair is cut by barber 1 given that you received a good haircut.

b.(5 points) Assume $\Pr(G) = \frac{16}{30}$. Find x .

c.(5 points) Assume that the barber who cuts your hair on the first visit will cut your hair on all subsequent visits. Calculate $\Pr(B_1|G_1 \cap G_2 \cap G_3 \cap NG_4 \cap G_5 \cap G_6)$, where G_j is the event that you received a good haircut on visit $j \in \{1, 2, 3, 4, 5, 6\}$.

d.(8 points) Suppose now that you have visited the barber shop only once and received a good haircut. Assume also that the barber who cut your hair on the first visit will cut your hair on the second visit. What is the probability that you will get a good haircut in your second visit (assume x is the value you found in part b).

4. (22 points)

a.(14 points) Write down the corresponding pdf and cdf of the random variable X graphed in Figure 4.1. (Remember to clearly state the relevant intervals.)

b.(8 points) Define now a new random variable Y , which can take the value $Y = X$ with probability 0.8 and $Y = 10$ with probability 0.2. Use your answer to part **a** to write the pmf/pdf and the cdf of Y . (Remember to clearly state the relevant intervals.)