

Exam 3
14.30 Fall 2004
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Instructions: This exam is closed-book and closed-notes. You may use a simple calculator. Please read through the exam in order to ask clarifying questions and to allocate your time appropriately. You **must show** all your calculations. You have 90 minutes to complete the exam (90 points). Good Luck!

1. **(21 points, 3 points each)** True, False, or Uncertain. Please avoid long explanations (but, no explanation no points).

- (i) Knowing the null distribution of a hypothesis test and the decision rule allows one to calculate the probability of types I and II error.
- (ii) Maximum likelihood Estimation results in unbiased and consistent estimators.
- (iii) Method of Moments Estimation results in unbiased and consistent estimators.
- (iv) $\hat{\sigma}_{MLE}$ is enough information to compute $\hat{\sigma}_{MLE}^2$.
- (v) An optimal hypothesis test is defined as the test that minimizes types I and II error.
- (vi) A minimum mean squared error estimator is defined as the most efficient estimator.
- (vii) The sample mean has always a normal distribution regardless of the underlying, or population, distribution.

2. (23 points) You have a random sample of size n from a $U[0, \theta]$: X_1, \dots, X_n . However, you do not observe X_i directly, but only a signal Y_i of the process. Whenever X_i is greater than 0.5, Y_i takes the value of 1. Whenever X_i is lower or equal than 0.5, Y_i takes the value of 0.

a.(5 points) Which is the pdf of Y_i ?

b.(5 points) Derive the method of moments estimator for θ .

c.(7 points) Derive the maximum likelihood estimator for θ .

d.(6 points) Assume that you still observe Y_i , but you do not know the cutoff value h , such that $Y_i = 1$ if $X_i > h$ and $Y_i = 0$ if $X_i \leq h$ (so, in the previous parts $h = 0.5$). Explain, but do not compute, how would you find $\hat{\theta}_{MLE}$.

3. (26 points) Assume that students' GRE results are normally distributed with mean (μ) 600 and standard deviation 60. A new studying method (*GRE+Plus*) has been developed and its authors claim that it improves the students' performance in the GRE. Assume you have data on the GRE score of 16 students that studied with *GRE+Plus*.

a.(7 points) Construct a 5% test of the null hypothesis that the *GRE+Plus* method does not affect the students' performance against the alternative hypothesis that it improves their performance. (Do not forget to show how you derive the values for the decision rule.)

b.(10 points) What is the power of the test at $u = 620$, at $u = 640$, and at $u = 680$? How would you interpret your results if the test fails to reject the null hypothesis? How does your answer change if the number of students in the sample increases?

c.(4 points) Explain, but do not compute, how your answer to **part a** changes if you do not know the variance of the population distribution.

d.(5 points) Up to here we have assumed that the *GRE+Plus* method either improves the students' GRE score or does not change it at all. Now you are told that it could also negatively affect the students' performance in the GRE. Given this information, write the new null hypothesis assuming the same alternative hypothesis as in **part a**. Which will be the size of this new test if you keep the same decision rule as in **part a**? Explain your answer.

4. **(20 points)** A herbalist is experimenting with a juice extracted from berries and roots that may have the ability to affect the level of human body's calcium absorption. Assume that past experience suggest that human body's calcium absorption averages 0.5 with variance 0.09. You observe the calcium absorption level of 400 people after they took the juice for 6 months.

a.(7 points) Construct a two-sided and 3%-size hypothesis test of the null hypothesis that the juice does not affect the calcium absorption levels. (Do not forget to show how you derive the values for the decision rule.)

b.(6 points) Assume that the average calcium absorption level in the sample is 0.539. Test your hypotheses and find the p-value.

c.(7 points) Assume now that the sample consisted of 40 individuals instead of 400 people, and that you want to be cautious because you do not want to claim to have a new drug when you really do not have one. What would you change, if any, from the test proposed in **part a**? Why? Do not compute the test again, just explain.

The End...

...and good luck with the rest of your exams!