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14.30 Introduction to Statistical Methods in Economics
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Problem Set #9

14.30 - Intro. to Statistical Methods in Economics

Instructor: Konrad Menzel

Due: Friday, May 8, 2009

Question One: Confidence Intervals

(Adapted from Bain/Engelhardt p. 384)

Consider a random sample of size n from a normal distribution, $X_i \sim N(\mu, \sigma^2)$.

1. If it is known that $\sigma^2 = 15$, find a 90% confidence interval for μ based on the estimate $\bar{x} = 25.3$ with $n = 16$.
2. Based on the information in (1), find a one-sided lower 90% confidence limit for μ . Also, find a one-sided upper 90% confidence limit for μ .
3. For a confidence interval of the form given by $(\bar{x} - \Phi^{-1}(1 - \frac{\alpha}{2})\frac{\sigma}{\sqrt{n}}, \bar{x} + \Phi^{-1}(1 - \frac{\alpha}{2})\frac{\sigma}{\sqrt{n}})$, derive a formula for the sample size required to obtain an interval of specified length, λ . If $\sigma^2 = 9$, then what sample size is needed to achieve a 90% confidence interval of length 2?
4. Suppose now that σ^2 is unknown. Find a 90% confidence interval for μ if $\bar{x} = 25.3$ and $s^2 = 15.21$ with $n = 16$.
5. Based on the data in (4), find a 99% confidence interval for σ^2 . Also show for $n = 14$. (Hint: What is the distribution of $\frac{n \cdot s^2}{\sigma^2}$? It's a χ_{n-1}^2 , which happens to approach the Normal distribution, but use the χ_{n-1}^2 for this part.)
6. Now use the Normal approximation to get a 99% confidence interval for σ^2 . Also show for $n = 14$. Does your estimate make sense for $n = 16$ and $n = 14$? Explain.

Question Two: Empirical Example

The following data are times (in hours) between failures of air conditioning equipment in a particular airplane: 74, 57, 48, 29, 502, 12, 70, 21, 29, 386, 59, 27, 153, 25, 232. Assume that the data are observed values of a random sample from an exponential distribution $X_i \sim EXP(\theta)$.

1. Find a 90% confidence interval for the mean time between failures, θ .
2. Find a one-sided lower 95% confidence limit for the 10th percentile of the distribution of time between failures.

Question Three: Hypothesis Testing Concepts

1. Define the null, H_0 , and alternative, H_a , hypotheses and explain their difference.
2. Write down the definition of a Type I error twice: first, mathematically and second, in words.
3. Now, write down the definition of a Type II error twice: first, mathematically and second, in words.
4. Fill in where “Type I error” and “Type II error” fall in the following box:

| | <i>Reject H_0</i> | <i>Fail to Reject H_0</i> |
|------------------------------|--------------------------------|--|
| <i>H_0 True</i> | | |
| <i>H_a True</i> | | |

5. In the United States, the criminal justice system claims that a suspect is “innocent until proven guilty.” Write down the equivalent null hypothesis and its complementary alternative for this statement. Further, define the Type I and Type II errors. Give your answer in a box as above in (4).

Question Four: Power Curves

Suppose that a sample of size one is taken from the PDF $f_Y(y) = \frac{1}{\lambda}e^{-\frac{y}{\lambda}}$, with $y > 0$, for the purpose of testing

$$H_0 : \lambda = 1 \text{ versus } H_1 : \lambda > 1.$$

The null hypothesis is rejected if $y > 3.20$.

1. Calculate the probability of committing a Type I error.
2. Calculate the probability of committing a Type II error when $\lambda = 1, \frac{4}{3}, \frac{3}{2}, 2, \frac{5}{2}, 3, \text{ or } 3.2$.
3. Let the Type II error be represented by β . Plot the values of $1 - \beta$ from (2) against λ and connect the dots. You just constructed the power curve for the test!
4. Sketch what you think the power curve for the test $H_0 : \lambda = 1 \text{ versus } \lambda \neq 1$ will look like if we reject the null hypothesis in the event that $y \notin [0.25, 1.75]$. In particular, will it be symmetric or skewed? Compute the probability of a Type II error if $\lambda = 0.25$ and 1.75 to corroborate your answer.

Question Five: Application of Hypothesis Testing

Commercial fisherman working certain parts of the Atlantic Ocean sometimes find their efforts being hindered by the presence of whales. Ideally, they would like to scare away the whales without frightening the fish. One of the strategies being experiment with is to transmit underwater the sounds of a killer whale. On the 52 occasions that this technique

has been tried, it worked 24 times (that is, the whales got lost). Experience has shown, though, that 40% of all whales sighted near fishing boats leave of their own accord anyway, probably just to get away from the noise of the boat and the bad smell.

1. Let p be the probability that a whale leaves the area after hearing the sounds of a killer whale. Test $H_0 : p = 0.40$ versus $H_1 : p > 0.40$ at the $\alpha = 0.05$ level of significance. Can it be argued on the basis of these data that transmitting underwater predator sounds is an effective technique for clearing fishing waters of unwanted whales?
2. Calculate the p - values for these data. For what values of α would H_0 be rejected?

Question Six: One v. Two Sided Hypotheses; Sample Variance

Suppose that babies' weights at birth are normally distributed with a mean of 7 pounds and a variance of 1 pound. A particular obstetrician is suspected of giving pregnant women poor advice on diet, which would cause babies to be 1 pound lighter on average (but still have the same variance). You observe the weight of $n = 10$ babies that he delivers. The mean weight of the 10 babies is 6.2 pounds.

1. Suppose you want to do a test of the null hypothesis that the obstetrician is not giving poor advice against the alternative hypothesis that he is. Write down the null hypothesis and alternative hypothesis mathematically.
2. Perform a 5% test of the null hypothesis.
3. Suppose you only knew the mean of the distribution of babies' weight, not the variance, but you did have an estimate of the variance, $s^2 = 1.5$. Perform a 5% test of the null against the alternative.
4. Would any of your answers above change if your alternative hypothesis was that the obstetrician was doing something to affect babies' weights, either negatively or positively? If so, adjust your p -values and conclusions accordingly.