

## Problem Set II

MIT (14.32)  
Spring 2007

due: March 8, 2007

A. From Wooldridge: B.10 (more on expectation), 2.4, 2.8, 2.9 (bivariate regression basics)

B. Additional problems

1. Consider  $E[Y|X]$  where  $X$  is a dummy variable that equals one with probability  $p$  and is zero otherwise. Prove that the CEF and the regression of  $Y$  on  $X$  are the same in this case. Do this by showing that for Bernoulli  $X$ :

$$\alpha = E(Y) - \beta E(X) = E[Y|X=0]$$

$$\beta = \text{COV}(X, Y) / V(X) = (E[Y|X=1] - E[Y|X=0])$$

2. The 14.32 web page contains a data set with observations on Log(Usual weekly earnings), Sex, Age and Education for respondents aged 25-29 in the March 1992 CPS.

a. Run PROC CONTENTS and PROC MEANS to verify the contents of the data set.

b. Use PROC TTEST to compare average log wages by sex for a subsample aged 40-49. Use PROC REG to prove the result in question B.1 above "by computer."

c. PROC TTEST calculates the standard error of differences in averages under two different assumptions about variances. State these assumptions in words. Which calculation corresponds to the one being used by PROC REG? Explain and check your answer.

C. Regression application.

For this problem, you will be provided with data on the price of wine (Chateau Latour) sold at a 1986 London auction. Wines sold were from vintage years 1882 to 1983. The same data set includes information on rainfall and temperature in the region in France where Chateau Latour grapes are grown.

The major determinant of wine quality is vintage (i.e., time passed since the wine was bottled.) This suggests that wine prices should increase with age. Let  $t$  be the year a bottle of wine is bottled and  $T$  be the year that it is sold at auction. When sold, the wine is  $\tau = T - t$  years old. Suppose the unknown price of a wine aged  $\tau$  is an increasing concave function  $f(\tau)$ . The present value of wine in the year it is bottled is therefore

$$v(\tau) = f(\tau) / (1 + r)^\tau,$$

where  $r$  is the discount (interest) rate used by investors to evaluate future prospects.  $f(0)$  is the price of the wine in the year it is bottled.

1. Like financial investors holding a treasury bond, vintners and wine investors can sell their wine now or hold it and sell it at a later date. One theory of asset pricing in financial markets is that market forces will set prices to *equalize the present value* of an asset sold on different dates. Why is this a compelling theory?

2. Show that equal present values at different vintages implies

$$(1) \quad \ln[f(\tau)] \approx r\tau + \ln[f(0)]$$

3. Use the data for this problem set and equation (1) to estimate the discount rate used by those who buy and sell Chateau Latour. Test whether  $r$  equals 3 percent. Plot the data and fitted values from your regression. Briefly discuss the fit.