

# 14.44/14.444 Energy Economics, Spring 2006

## Problem Set 2

Due Friday February 23, 2006 in class or Arthur Campbell's mail folder

Late problem sets are not accepted

This problem set reviews your knowledge of multivariate regression analysis. It requires you to answer questions related to the article on the reading list "Evidence of a shift in the short-run price elasticity of gasoline demand" by J. Hughes, C. Knittel and D. Sperling. Some of you may also find the article "An introduction to regression analysis" by A. Sykes, also on the reading list, is a useful reference for the material.

1. (60) In section 2 the authors estimate the following demand equation:

$$\ln G_{jt} = \beta_0 + \beta_1 \ln P_{jt} + \beta_2 \ln Y_{jt} + \varepsilon_j + \varepsilon_{jt}$$

where  $G_{jt}$  is per capita gasoline consumption in gallons in month  $j$  and year  $t$ ,  $P_{jt}$  is the real retail price of gasoline in month  $j$  and year  $t$ ,  $Y_{jt}$  is real per capita disposable income in month  $j$  and year  $t$ ,  $\varepsilon_j$  represents unobserved demand factors that vary at the month level and  $\varepsilon_{jt}$  is a mean zero error term.

- a) (10) What have the authors assumed about the price elasticity of demand when they wrote down the demand equation in this form? Remember the price elasticity of demand  $E_p = \frac{\partial G}{\partial P} \frac{P}{G}$ .

Ans: The assumption made by the authors is that consumer demand for petrol has a constant elasticity of demand at all points along the demand curve.

- b) (10) Go to table 1 in the appendix, now assuming the authors have obtained unbiased estimates of the parameters  $\beta_0, \beta_1$  and  $\beta_2$  what do they mean? (eg. the coefficient  $\beta_1$  is -0.335 in the period 1975-1980, this represents...)

Ans: The coefficient  $\beta_1$  is the price elasticity of demand, in the period 1975-1980 it is -0.335, this means that for a 1% increase in price in this period demand will fall by 0.335%, similarly in the period 2001-2006 it is -0.042 so for the same price change demand falls by 0.042%.

The coefficient  $\beta_2$  is the income elasticity of demand in the period 1975-1980 it is 0.467, this means that for a 1% increase in income in this period demand will increase by 0.467%, similarly in the period 2001-2006 it is 0.530 so for the same change in income demand increases by 0.530%.

The coefficient  $\beta_0$  is the constant in the demand specification. This scales the demand relation in such a way (holding demand elasticity constant) larger values of  $\beta_0$  indicate larger per capita demand for the same price across different time periods. The value of  $\beta_0$  decreases from -0.615 to -1.697 from 1975-1980 to 2001-2006. This represents a fall in per-capita gasoline consumption which is consistent with the data presented in Figure 1 of the paper.

- c) (10) Interpret the values of the monthly unobserved demand factors ( $\varepsilon_j$ )? What are these relative to? What can you say about the yearly pattern of gasoline demand from these coefficients?

The values of the monthly unobserved demand factors can be interpreted in much the same way as the constant term  $\beta_0$ . That is greater values of the coefficient relative to other time periods or months indicates relatively higher demand. The yearly pattern of gasoline demand is characterized by a peak during August and a low during February and intermediate levels inbetween. The monthly factors are relative to the level in December which is normalized to 0 by including the constant term and excluding the month dummy for December.

- d) (10) From the information presented in this table calculate the appropriate t-statistics for each of the  $\beta$ 's to test if it is different from 0. You will need the standard errors for each coefficient which are presented in brackets below the respective coefficient value in the table. For instance the standard error for the coefficient  $\beta_1$  in the period 1975-1980 is 0.024.

T-statistics are calculated by dividing the value of the co-efficient by the standard deviation. They are used to test whether a coefficient is significantly different from 0. For example the first entry in the table below is calculated by  $\frac{-0.615}{0.929} = -0.662$ .

	1975-1980	2001-2006
$\beta_0$	-0.662	-2.891
$\beta_1$	-13.958	-4.667
$\beta_2$	4.865	9.138

- e) (10) What do the \*\*\* next to some of the entries in the table indicate? How are they related to the t-statistics you calculated?

This indicates that the coefficient is significant at a level of 1%. The t-statistics are used along with the student-t distribution to determine at what level the coefficient is significant

- f) (10) The table presents the adjusted R-squared statistic for the two regressions. What does this number mean? If we calculated the unadjusted R-squared values, can we say whether these are larger or smaller than the adjusted R-squared values of 0.84 and 0.94 in this table?

The adjusted R-squared values represent the percentage of the variation in per-capita consumption is captured by the regression adjusted for the number of variables in the regression. The unadjusted R-squared values will be greater than the adjusted values.

2. (30) In table 2 and table 3 in the appendix, two alternate specifications for the demand equation are compared with the original double-log model.

- a) (10) Under the linear specification for the period 1975-1980 the coefficient on the Price variable is -7.252. What is the implied elasticity of demand, assuming the linear model, if during a June month per capita demand was 40 gallons, and price was \$1.70?

$$E_p = \frac{\partial G}{\partial P} \frac{P}{G} = -7.252 \times \frac{1.7}{40} = -0.308$$

- b) (10) What is the implied elasticity of demand if during July demand is 5 gallons higher (due to a month specific effect) and price is the same?

$$E_p = \frac{\partial G}{\partial P} \frac{P}{G} = -7.252 \times \frac{1.7}{45} = -0.274$$

- c) (10) Under the linear demand specification the demand elasticity varies within each period. Therefore the authors calculate an "average" elasticity of demand across each period. Do you think a time weighted or quantity weighted average is more reasonable and why?

A quantity weighted average seems more reasonable since we are dealing with elasticities. In terms of the amount of quantity demanded across the entire period and how this is affected by the overall price level then additional weight should be attached to periods/months during which there was more demand. In the extreme case where only a couple of gallons are demanded in a month the very small changes which occur during this month should not have as large an impact on the overall demand elasticity as say a month with very large gasoline demand.

3. (20) In section 3.3 the authors' specify a model where there is an interaction between price and income

$$\ln G_{jt} = \beta_0 + \beta_1 \ln P_{jt} + \beta_2 \ln Y_{jt} + \beta_3 \ln P_{jt} \ln Y_{jt} + \varepsilon_j + \varepsilon_{jt}$$

where  $\beta_3$  is the coefficient on the interaction term.

- a) (10) Derive the price elasticity of demand using this model specification. You should get  $E_p = \beta_1 + \beta_3 \ln Y_{jt}$ . What would be the expression for the income elasticity of demand?

Differentiating the demand relationship with respect to  $P_{jt}$ :

$$\frac{1}{G} \frac{\partial G}{\partial P} = \frac{\beta_1}{P_{jt}} + \frac{\beta_3 \ln Y_{jt}}{P_{jt}}$$

which can be rearranged to get

$$E_p = \frac{P}{G} \frac{\partial G}{\partial P} = \beta_1 + \beta_3 \ln Y_{jt}$$

similarly differentiating with respect to  $Y_{jt}$

$$\frac{1}{G} \frac{\partial G}{\partial Y} = \frac{\beta_3}{Y_{jt}} + \frac{\beta_3 \ln P_{jt}}{Y_{jt}}$$

once more rearranging to get

$$E_Y = \frac{Y}{G} \frac{\partial G}{\partial Y} = \beta_2 + \beta_3 \ln P_{jt}$$

- b) (10) Interpret the values of  $\beta_3$  presented in table 7 in the appendix. What do these tell you about the effect of income on the price elasticity of demand? How has this changed from the 1975-80 period to today?

$\beta_3$  is -2.879 during 1975-80 and -1.014 during 2001-06. This tells us that the interaction between price and income has become weaker. That is % changes in price and income have relatively less impact on the elasticity of demand wrt the other variable. The negative sign indicates that increases in income decrease the elasticity of demand wrt price, because price elasticities are normally negative this in fact makes demand more elastic. On the other hand increases in prices decrease the elasticity of demand wrt income which is positive here and thus makes it more inelastic.

4. (40) (14.444 students only) In section 3.2 the author's find that when the simultaneity bias is accounted for in the period 2001-2006 the estimate for the elasticity of demand changes from -0.042 to -0.077 and this change is statistically significant. The authors' conclude that this is encouraging and that the effects of simultaneity are small relative to other factors. Give a brief (1/2 a page max.) critique of the authors' methodology for addressing the simultaneity bias.

So in this section I was just after a bit of a discussion of both the instrument the authors' used to address the simultaneity bias and then what some of the assumptions are that would make their approach valid in the event that they had not introduced a simultaneity bias through using a regression of quantity directly on price. In particular this assumes that variation observed in the data is not coming from demand side shocks rather there are supply side shocks which provide the variation in prices and quantities which allow us to identify the demand elasticity.