14.661: Recitation 11

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These notes borrow heavily from Daron Acemoglu's labor economics lecture notes.

1 Education as a Signal: Spence (1973)

We have spent a lot of time studying the effect of schooling on earnings. In the classic human capital model, schooling increases earnings because it adds' to workers productivity. An alternative theory of education (known as "signaling," and due to Spence (1973)) argues that education may add little or nothing to workers' productive capacities. The key to this model is that productivity is not observable to employers, so the "good" workers would like to somehow signal their quality to employers. If obtaining education is cheaper for better workers (perhaps because it's more difficult for less productive workers to attain high levels of education), this can lead to an equilibrium where people get educated despite the fact that education does not add to productivity.

1.1 Basic signaling model

The basic signaling model has the following elements:

- There are two types of workers: H's (fraction λ) and L's (fraction $1-\lambda$). Firms do not observe workers' types.

- The two types of workers have different productivites, y_H and $y_L < y_H$.

- There are two possible schooling levels, $s \in \{0, 1\}$. Schooling levels are observable to firms.

- The two types of workers have different costs of obtaining schooling: $c_L > c_H$

- The timing of the game is as follows: First, workers choose education, and firms observe it. Then, firms make wage offers.

- Firms compete a la Bertrand, so that workers are paid their expected productivities given firm beliefs

1.2 Equilibrium concept

This is a dynamic game of incomplete information, so the standard equilibrium concept is Perfect Bayesian Equilibrium (PBE). This equilibrium has the following features:

1. All players maximize their payoffs given other players' strategies and their own beliefs

2. Beliefs are "correct" (that is, they are consistent with Bayes' rule where applicable, and can be anything where it is not applicable).

We will see how this works below. Signaling games often have many PBE.

1.3 Separating Equilibrium

First, let's look for an equilibrium where the high types get schooling and the low types don't. Firm beliefs must be

$$Pr\left[H|s=1\right]=1,\ Pr\left[H|s=0\right]=0$$

since these are the actions taken by each type in equilibrium. Wage offers are therefore $w(1)=y_H$, $w(0)=y_L$. Given this wage schedule, H's will want to get $s=1$ if

$$y_H - c_H \geq y_L$$

and L's won't get schooling if

$$y_H - c_L \leq y_L$$

so this equilibrium is sustainable iff

$$y_H \in [y_L + c_H,\ y_L + c_L]$$

This is possible since $c_L > c_H$, though it is not necessary.

1.4 Pooling Equilibrium

Next, let's look for an equilibrium where workers pool on $s=0$. Here, firm beliefs must be

$$Pr\left[H|s=0\right]=\lambda$$

since all workers choose $s=0$ and the population fraction of H's is λ. However, since in this equilibrium no one chooses $s=1$, firm beliefs given $s=1$ are unrestricted. Let's specify $Pr\left[H|s=1\right]=0$ (that is, firms believe that educated workers are definitely low types). Wage offers will therefore be

$$w(0) = \lambda y_H + (1-\lambda)y_L \equiv \bar{y},$$

$$w(1) = y_L$$

Workers will not deviate if

$$\bar{y} \geq y_L - c_H$$

$$\bar{y} \geq y_L - c_L$$

But since $\bar{y} > y_L > y_L - c_H > y_L - c_L$, both are necessarily satisfied, and this equilibrium always exists.

1.5 Equilibrium Selection

Both the separating equilibrium and the pooling equilibrium are PBE (when the separating equilibrium exists). However, note that the pooling equilibrium is more "efficient" in the sense of maximizing total surplus; in both equilibria, all workers are employed, but in the separating equilibrium high types burn resources to signal their productivities. Schooling is pure social waste in the signaling model. Note, however, that moving from the signaling equilibrium to the pooling equilibrium may not be a Pareto improvement since it lowers wages for high type (though it can be).

But which equilibrium should we expect? Note that in the pooling equilibrium, we made a somewhat strange assumption about firms' off-equilibrium beliefs – they believed that all deviators were low types, for whom deviations are more costly. Since these beliefs are not restricted, this is a PBE, but we might think that such beliefs are "unreasonable" in some sense. Cho and Kreps' "Intuitive Criterion" (IC) is an equilibrium refinement that gives a formal account of what it means for beliefs to be unreasonable. For a particular equilibrium to fail the IC, there must be two types, T and T', for whom the following conditions are satisfied for a particular deviation:

1. "Type T wouldn't do this:" Type T is doing better in equilibrium than he could do by deviating for ANY beliefs other players could have. In the pooling equilibrium of the signaling game, type L could never do better by getting schooling iff $\bar{y} > y_H - c_L$; even if firms believe deviators are high types, schooling is still not worth it.

2. "Type T' would do this:" As long as other players don't believe that type T would deviate, it is always strictly better for T' to deviate. In the pooling equilibrium of the signaling game, if firms don't believe that type L would get schooling, then it is always better for high types to get schooling iff $y_H - c_H > \bar{y}$

The IC is usually motivated by a speech given by type T' to other players after deviating. He could say: "You should believe that I am not type T. No matter what you were going to think, he would never have deviated, but as long as you're reasonable and don't think I am T, I would. This is why I did it!" This may or may not make sense; such speeches are not part of the allowed actions in the game.

If $y_H \in (\bar{y} + c_H, \bar{y} + c_L)$, the pooling equilibrium fails the IC. If

$$y_H \in (\bar{y} + c_L, y_L + c_H],$$

then the separating equilibrium exists and the pooling equilibrium fails the IC.

According to the IC, we might therefore expect to see the separating equilibrium under some plausible conditions, despite the fact that it produces less total social surplus.

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