

# Graduate Labor Economics Review Exercise

Labor Economics  
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Due in first recitation

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1. A consumer chooses between two goods,  $x_1$  and  $x_2$ , with prices  $p_1$  and  $p_2$ , given income  $y$ , so as to maximize utility.
  - (a) Graph the consumer's problem in  $x_1$  and  $x_2$  space. Show the consumer's optimal choice.
  - (b) Give a graphical decomposition of the income and substitution effects from a change in  $p_1$  on the demand for  $x_1$ .
2. Assume the consumer's utility function is given by:

$$u(x_1, x_2) = \alpha \ln(x_1 - \gamma_1) + (1 - \alpha) \ln(x_2 - \gamma_2),$$

where:  $0 \leq \alpha \leq 1$ ,  $x_1 > \gamma_1 \geq 0$ ,  $x_2 > \gamma_2 \geq 0$ .

(This is called a Stone-Geary utility function. The parameters  $\gamma_1$  and  $\gamma_2$  are sometimes thought of as "subsistence" levels of consumption, below which utility is not defined.)

- (a) What is the marginal rate of substitution (MRS) for this utility function?
- (b) Derive the first-order conditions for the optimal choices of  $x_1$  and  $x_2$  as a single equation involving the MRS. What is the graphical interpretation of this condition?
- (c) Derive the uncompensated ("Marshallian") demand function for  $x_1$  for this consumer. Use Roy's identity to confirm this.
- (d) Show that the consumer's *expenditure* on  $x_1$  is a linear function of  $p_1$  and  $y$ . For example, for good  $x_1$ , we have

$$p_1 x_1 = p_1 \gamma_1 + \alpha (y - p_1 \gamma_1 - p_2 \gamma_2).$$

For this reason, the Stone-Geary utility function is sometimes called the "Linear Expenditure System." Interpret this equation.

- (e) What is the cost function for this problem?
  - (f) Derive the compensated ("Hicksian") demand function for  $x_1$  for this consumer using Shephard's lemma. What is the graphical interpretation of a compensated price response? What is the Slutsky equation for this problem?
  - (g) Show the Slutsky equation holds for Stone-Geary preferences.
3. Empirical researchers often work with the Cobb-Douglas production function:

$$Q = \gamma L^\alpha K^\beta; \quad \alpha > 0, \quad \beta > 0, \quad \gamma > 0,$$

where  $L$  is labor,  $K$  is capital, and  $Q$  is the quantity of output.

- (a) What must be true for this production function to exhibit constant returns to scale?
  - (b) Assuming you have the necessary data, describe at least two ways to construct a statistical test for constant returns to scale in production.
  - (c) Suppose you have data on a panel of firms. Explain how to use this data to control for an unobserved but time-invariant variable that captures managerial efficiency.
4. What is the present value of  $x$  dollars/year for  $t$  years with interest compounded annually? Derive the equivalent rate under continuous compounding. Suppose a fixed cost of  $C$  paid up front yields a stream of benefits  $B$  starting in 10 years. Define the internal rate of return for this project and explain how to use this rate to make investment decisions.

5. Suppose that students' demand for cellular data service is a linear function of the price of GBs transferred and the annual income of college students. Make up some notation for a demand equation that reflects this assumption. Use this equation to compute the price-elasticity of demand for data service as a function of demand parameters and price, income, and quantity sold.
6. Regression basics
  - (a) Regression anatomy: Explain how to calculate a multivariate regression coefficient using a two-step procedure where the second step is a bivariate regression.
  - (b) The omitted variables bias formula: What is the relationship between the regression coefficient from a bivariate regression of wages on schooling (a "short regression") and the coefficient on schooling when the equation also includes test scores (a "long regression")? Generalize this to the case where both the short and long models include other covariates.
7. Two-stage least squares: Show that 2SLS is an IV estimator. What is the instrument?
8. Grouping and IV: show that an equation estimated using group averages (e.g., average hours worked by city on average hourly earnings by city) is the same as 2SLS. What are the instruments?
9. Suppose the probability a woman works is modeled using a latent-index model such that

$$y_i = 1(X_i' \beta > \varepsilon_i),$$

where  $y_i$  is employment status,  $X_i$  is a vector of personal characteristics, and  $\varepsilon_i$  is  $N(0, \sigma^2)$  and independent of  $X_i$ . Write down the likelihood function and derive the FOCs. What is identified? Propose a weighted least squares procedure that is asymptotically equivalent to the MLE.

10. Derive the formula for the standard error of a difference in means. Show that if the outcome variable is homoskedastic, the optimal experimental design for estimating a difference in means has the proportion treated equal to 1/2.
11. Consider two random variables,  $Y$  and  $X$ . Show that the variance of  $Y$  can be written as the variance of the conditional mean function,  $E[Y|X]$ , plus the average variance of  $Y$  conditional on  $X$ . This is called the analysis of variance (ANOVA) identity. Prove that ANOVA holds for samples as well and interpret the pieces.

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